Does releasing foreign investment bounds improve the performance of pension funds?*

*Miguel Víctor Chirinos Grados***
*Fernando José Cadillo Vásquez****

* doi: 10.11144/Javeriana.cao28-50.drfi. Este artículo se recibió el 18/08/2014 y se aprobó el 30/05/2015. Sugerencia de citación: Chirinos G., M. V., and Cadillo V., F. J. (2015). Does releasing foreign investment bounds improve the performance of pension funds? *Cuadernos de Administración*, 28 (50), 61-89. http://dx.doi.org/10.11144/Javeriana.cao28-50.drfi

** Magíster en Finanzas de la Universidad del Pacífico de Lima, Perú, 2013. Docente de cátedra Finanzas de la Facultad de Ciencias Económicas de la Universidad Nacional de Bogotá, Colombia.
Correo electrónico: mvchirinosg@unal.edu.co

*** Licenciado en Administración de Empresas y en Contabilidad de la Universidad del Pacífico de Lima, Perú, 2005. Docente del Departamento Académico de Finanzas de la Universidad del Pacífico, Lima, Perú.
Correo electrónico: cadillo_fj@up.edu.pe

Does releasing foreign investment bounds improve the performance of pension funds?

Abstract

This article evaluates the regulator's proposal to increase the bounds on foreign investment of the Peruvian Pension Funds. The subject is analyzed from two perspectives: the first determines whether higher investment levels in foreign markets improve the diversification of the investment portfolio; to specify the co-relation level we make use of the copula model. The second perspective analyzes the effects caused by the clashes generated in the foreign markets through the extreme value theory. The results suggest, as a policy recommendation, to reduce the levels of investment on shares and foreign investment in the investment portfolios of the pension funds.

Keywords: Contagion, copula, extreme value theory, portfolio.
JEL Classification: G11, G23, G28, G32

¿La liberación de los límites de inversión en el extranjero mejora el desempeño de los fondos de pensiones?

Resumen

Este artículo evalúa la propuesta del regulador para incrementar los límites de inversión en el extranjero de los fondos de pensiones peruanos. El tema es analizado desde dos perspectivas: la primera determina si mayores niveles de inversión en los mercados externos mejora la diversificación de la cartera de inversión; para precisar el grado de correlación hacemos uso del modelo de cópulas; la segunda analiza los efectos de los choques generados en los mercados externos mediante la teoría de valores extremos. Los resultados sugieren, como recomendación de política, reducir los niveles de inversión en acciones y en inversión en el exterior en la cartera de inversiones de los fondos de pensiones.

Palabras clave: contagio, cópulas, teoría del valor extremo, cartera de inversión.
Clasificación JEL: G11, G23, G28, G32

A liberação dos limites para investimento no exterior melhora o desempenho dos fundos de pensão?

Resumo

Este artigo avalia a proposta do regulador para aumentar os limites de investimento no exterior dos fundos de pensão peruanos. O tema é analisado de duas perspectivas: a primeira determina se maiores níveis de investimento nos mercados externos melhora a diversificação da carteira de investimento; para determinar o grau de correlação, usa-se o modelo de cópulas; a segunda analisa os efeitos dos choques gerados nos mercados externos mediante a Teoria dos Valores Extremos. Os resultados sugerem, como recomendação de política, reduzir os níveis de investimento em ações e investimento no exterior na carteira de investimentos dos fundos de pensão.

Palavras-chave: contagio, copula, teoria de valores extremos, carteira de investimentos.
Classificação JEL: G11, G23, G28, G32

Introduction

Both pension funds managers and regulators could expect that relaxing the limits of foreign investment improves the management of portfolio, but really does releasing foreign investment bounds improve the performance of pension funds? This paper analyzes the diversification effect such releasing from two point of view: Pearson correlation and extreme events.

The analysis is made in five sections. The first one is the antecedents, within such section, Pereda's (2007) work is the oldest analysis; this author asserts that lower bounds in foreign investment improves the return of pensions funds. The next paper analyzed is Ortiz, Chirinos and Hurtado (2010), this paper extended Pereda's (2007) work and both explain and assessment stylizes facts found in the first cited research. Ortiz *et al.* (2010) uses the zero-beta model for estimating the risk-free rate. In a later paper Chirinos and Ortiz (2013) evaluate the performance without need a risk-free rate; they found that lowers levels in equities investments improves the pension funds return. The previous papers were made under the assumption of normally distribution returns; the evidence show episodes of contagious and cluster volatility (ARCH processes), Chirinos (2013) analyzes this problems for Latin American countries. The later paper established the path that we will follow.

The second one section studies the impact of cluster volatility in the lineal correlation coefficient. Chirinos (2013) assumed symmetrical ARCH processes; exists evidence that indicates that such processes are asymmetrical. Even more, the correlation coefficient are changing through the time; from this perspective is adequate implement the Dynamical Correlation Coefficient (DCC). In this section are modeled –the univariates series– as asymmetrical ARCH processes, then are jointed through copulas technique. The elliptical family distributions are estimated. The third section is dedicate to the extreme event. There is evidence that the transmission of extremes shocks are different in one each tail; then, each tail is adjust individually. The fourth section makes the evaluation of the releasing the foreign-investment bounds. In the fifth and last section, we show the conclusions.

1. Antecedents

In 2010, the *Superintendencia de Banca, Seguros y Administradoras Privadas de Fondos de Pensiones* (SBS) – the Peruvian financial sector regulator – announced its decision to raise the foreign investment bounds in pension funds portfolios[1]. With the new limits, the managers would be able to invest up to 50% of the total fund value in foreign instruments. At the moment of its conception, in June 1993, the private pension fund system only allowed funds to allocate up to 5% of their total investment in foreign assets. Successive norms raised the bounds up to 10% and 20%. In fact, the "operative" bound, authorized directly by the Central Bank, remained fixed at 7.5% until May 2000; and step up to 30% after that date[2].

There is a handful of studies on the effects of foreign investment restrictions in the Peruvian market. Rivas-Llosa and Camargo (2002), using daily data from April 02, 2002 to September 25, 2002, estimated the efficient investment frontier with 265 assets. In a particularly short paper[3], the authors recommend an increase in the foreign investment bounds to 9.81%. This constituted the only ex-ante analysis; in June 2003 the limit stepped up to 9% and in April 2004 raised to 10.5%. Pereda (2007) studied the impact of the restrictions on foreign investment in pension funds using monthly data from January 1994 to December 2004. The author's main conclusion is "that in total they would have had a lower potential returns in the period analyzed by approximately 2.5 percentage points annual average, from which 1.9 percentage points would be attributable to limits imposed to pension funds investment"; the return lost is measured as the "vertical distance between the efficient frontier's curve with and without restrictions". However, "in the analyzed period the Pension Funds could have obtained greater return for their investment, with the same level risk as average portfolio". Ortiz *et al.* (2010) used Pereda's data and estimated the efficient frontier with short-selling position. Additionally, the authors estimated the zero-beta portfolio return introduced by Black (1972)[4]. The zero-beta portfolio expected return is an estimated of risk free rate which in the case of the Peruvian economy, the estimated rate is 11.87%. Using this value, Ortiz *et al.* (2010) explain the Pereda's (2007) findings: Several assets, such as Government Bonds, Foreign Bonds, Bank Account in foreign currency and others shouldn't be in the optimal

1 The decision was announced in interview given to Agencia Andina by the Superintendent–Adjoin, in November 16th, 2010.

2 The private system was created through *Decreto Ley* 25897, November 28, 1992. The regulation was emitted in December of 1992, *Decreto Supremo* N° 206-92-EF/90.

3 The paper has only two pages. We think the quality of a paper is not dependent of its extension; however we want to remark the difficulty we face in reproducing its results.

4 See Ortiz *et al.* (2010), and their references, for a discussion about restrictions in CAPM.

portfolio. These assets have returns lower than the risk free rate; that is, their return is less than 11.87%.

In addition to previous results, Ortiz *et al.* (2010) find that if the bounds for fixed income would be raise and the ones for equity reduced, the risk for the average return would have fallen from 10.0% to 3.8%; with the new bounds for investment, the average level risk (10%) would have been impossible to attain. In order to quantify the impact of the restrictions on investment these authors used a measure that takes into account the level risk: the Sharpe ratio. They find that the legal restrictions caused a decrease in the average return of 1.7%, but the managers generated an even larger decrease in the return average of 3.1%. The authors show that if the bounds are fixed as in the proposed simulation and we take the average risk of the pension funds as given, the average return would have been 17.8%; 3.7% higher than the return got by the fund managers.

The papers cited have a limitation: they need an estimated risk-free rate. Rivas-Llosa and Camargo (2002) fix it arbitrarily in 2.5%; Pereda (2007) ignores it; while Ortiz *et al.* (2010) estimated the expected return of the zero-beta portfolio as risk-free rate. The return estimated was 11.9%. Chirinos y Ortiz (2013) evaluated the performance of the pension funds managers without the necessity of either an ex-ante efficient portfolio or a risk-free rate by applying the Portfolio Change Measure introduced by Grinblatt and Titman (1993), which is a measure is based on portfolio recomposition.

In 2005, the SBS authorized the implementation of a multi-fund system created by the Law 27988, enacted in 2003; since the application of the new system, managers moved from a common (only one) fund to invest in three funds with different risk levels. These were denominated “Capital Preservation” (low risk), “Mixed Fund” (medium risk) and “Appreciation of Capital” (high risk). The limits for investment in equity were set at 10%, 45% and 80% respectively, compared to the 35% limit under the previous standard.

Chirinos and Ortiz (2013) analyzed the pension funds performance, the analysis is extended to December 2009; they found that, over the period following the changes, the performance was “poor” despite a higher level of risk. The worst performance in returns is the year 2008. If the evaluation takes account the higher levels of risk, the result for 2008 seems amazing: Managers strategies enabled gain 2.95% in the year that the funds had have the worse performance. In fact, the results are not surprising: In this year, managers decreased the portfolio weights in equities and, consequently, got higher yields; that is, the losses decreased. This is one of the key findings in Ortiz *et al.* (2010) and Pereda (2007): the lowers level risk becomes increased profitability.

The papers cited have another problem: they assume that the returns are normally and identically distributed. Mandelbrot (1963, p. 418) observed that the prices follows an ARCH process or "in other words, large changes tend to be followed by large changes –of either sign– and small changes tend to be followed by small changes". Alexander (2001, p. 63) stated that it was with the ARCH model introduced by Engle (1982) "that useful models of volatility clustering were developed", Bollerslev (1986) generalized the ARCH model.

To our knowledge, there is no document that studies these problems for the case of the Peruvian Pension Funds. The closest is the paper by Chirinos (2013)[5] which analyzes the stock market of Lima and others Latin American countries. This paper is a precedent, but has a limitation: it uses ARCH as a symmetric process. There is a vast literature about the asymmetry of shocks in the financial markets; Campbell, Koedijk and Kofman (2002) stated that "large negative returns from international equity markets tend to coincide much more frequently than would reasonably be expected from the unconditional return correlation". Arouri, Jawadi and Nguyen (2010, p.15) assert that correlations in international markets tend to increase in times of financial turbulence and crisis. Patton (2006) finds that, in the currency markets, the exchange rates are more correlated when they are depreciating than when they are appreciating. Chan-Lau, Mathieson and Yao (2004, p. 389) concluded that, for the equity markets, (a) bear market contagion is stronger than bull market contagion; (b) the emerging markets stocks are mostly affected by contagion with the United States of America and the United Kingdom; (c) Latin American is the only region where the number of dependent country-pairs has increased during the periods of bear markets contagion. Gençay and Selçuk (2004) study contagion in emerging markets, this paper include Argentina, Brazil and Mexico in the analysis; stated that "results indicate that daily return distribution have different moment restrictions at their left and right tails".

In view of the limitations of the current literature, the present paper studies the raise of the foreign investment bounds from two perspectives: the first one is the construction of an optimal portfolio, for this objective will make use of copulas; the second approach studies the effects of foreign-originated shocks in the local stock market by using the

[5] Gupta and Jithendranathan (2008) analyze the diversification of Australian investor with Mature and Emerging Markets. Found that average correlation with Peru was 0.441, the maximum was 0.594.

extreme value theory. With this analysis we expect shed some lights about if releasing the foreign investment bounds does improve the performance of pension funds.

2. The optimal portfolio

This section estimates the optimal portfolio using weekly data. The analysis is restricted to the stocks markets from Peru, United States of America (USA) and United Kingdom (UK); these countries were selected because the investments of PPF in British Pounds and US Dollars sum up to 41.3% of value of pension funds. The data corresponds to weekly return since December 31 of 1991 to December 31 of 2010. Returns are calculated every 7 days, the crude data is daily-index in US dollars calculated for the three countries. Before calculating the copulas, we show the correlation coefficients; after that, we calculate the ARCH/GARCH process for the returns. Once we obtain the marginal distributions, we estimate the multivariate distribution.

Table 1

Basic Statistics, weekly returns

	Peru	UK	USA
Mean	0.0074	0.0012	0.0016
Standard Deviation	0.0407	0.0238	0.0237
Kurtosis	6.6607	8.4659	5.9847
Skewness	0.5165	-0.5071	-0.5120
Minimum	-0.2925	-0.2105	-0.1820
Maximum	0.2192	0.1341	0.1203
Low (0.5%)	-0.0906	-0.0714	-0.0778
High (0.5%)	0.1741	0.0780	0.0729
IQR	0.0391	0.0257	0.0251
Jarque-Bera Test (*p*-value)	1956.3 (0.000)	3133.4 (0.000)	1587.2 (0.000)

Source: Own elaboration.

The basic statistics are shown in Table 1, the returns are weekly; Low and High represent the percentiles 0.5 and 99.5%, respectively; when the distribution is normal, this values should be within 2.58 standard deviation measure from the mean. IQR represent the difference of first and third quartile, which is considered more stable dispersion index than standard deviation when there is extreme value observations; that is because the IQR is not affected by changes in the lower and upper 25% of the data. The sample kurtosis are positives for the three countries, which varies from 6.0 (USA) to 8.5 (UK); kurtosis indicates the extent to which probability is concentrated in the center and especially the tails of the distribution. The weekly rates of return are far from being normally distributed; based on the sample kurtosis estimates, we are able to claim that the return distributions in all the markets are fat-tailed. The sample skewness shows that the weekly returns in all countries have an asymmetric distribution; it is positive for Peru and negative for UK and USA, negative skewness indicate that the negative tail extends more the negative values than the positive ones.

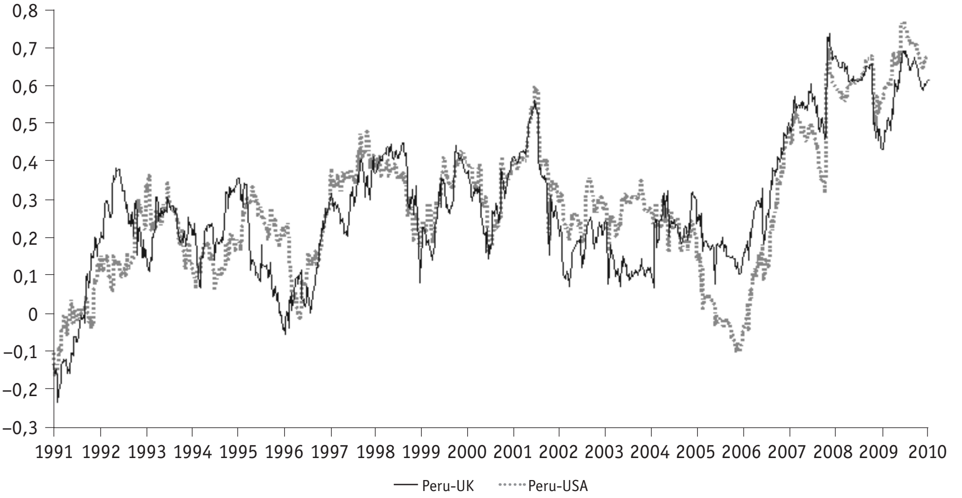

Figure 1. Correlation Coefficients, Peru with UK and USA. Weekly data

Source: Own elaboration.

Figure 1 shows the rolling (12 month) correlation coefficients for weekly returns in local currency, the analyzed period is 1991-2010; there is evidence that these coefficients aren't constants.

As it can be seen, in the last years the correlation coefficients between Peru and UK, in one hand, and Peru-USA, in the other, have been growing. This evidence suggests that the bounds for foreign investment should be re-weighted to lower levels. That is, when the diversification had lower effects on the portfolio risk, the supervisor decided to step up the bounds for foreign investment.

There is evidence that correlation change through time and from this perspective, it is more appropriate to analyze it with the Dynamic Conditional Correlation (DCC) introduced by Engle (2002); still more, as is it shown in Table 1, the returns are fat-tail and asymmetric. For the empirical work, we apply the methodology of Jondeau and Rockinger (2006) and the probability density function proposed by Hansen (1994); the implementation follows Vogiatzoglou (2010).

The marginal distributions are estimated from the following system of equations (1)-(3), which capture the features discussed previously; that is, fat-tail, skewness and asymmetries.

$$r_{i,t} = \mu_0 + \sum_{j=1}^{k} \mu_j r_{i,t-j} + e_{i,t} \tag{1}$$

$$e_{i,t} = \sqrt{h_{i,t}\varepsilon_{i,t}} \quad \varepsilon_{i,t} \sim SkT(v,\lambda) \tag{2}$$

$$h_{i,t} = \omega_{i,t} + \alpha e_{i,t-1}^2 + \beta h_{i,t-1} + \Upsilon e_{i,t-1}^2 \bullet 1_{\left[e_{i,t-1}<0\right]} \tag{3}$$

Table 2 shows the estimated parameters for GARCH (1,1) processes, Equations (1)-(3); the value in brackets is the *t*-Student test. The returns follow ARCH processes with autoregressive parameters in the mean for Peru and USA; UK has a constant expected return, the criteria for selection between autoregressive or constant in mean was the likelihood ratio. Equations (1)-(3) were estimated for each return; γ captures the asymmetric shocks, ν represents the freedom degree and λ the parameter of skewness of *t*-Student distribution proposed by Hansen (1994). The asymmetric is captured by the model of Glosten, Jagannathan and Runkle (1993), the indicator has value one when the shocks of the previous period were negative.

The next step addresses the joint distribution, which may be normal or another one, such as the *t*-Student. If the joint distribution were normal, the lineal correlation coefficient would get the dependence parameters. In this paper, we will estimate the family of elliptical joint distributions; for this family the correlation coefficient describe the dependence. With the evidence shown in Figure 1, the dynamical correlation coefficient introduced by Engle (2002) will be estimated; an alternative is to follow Tse and Tsui (2002). Table

3 shows the results for the static *t*-Student, dynamical Normal and dynamical *t*-Student. We won't study the selection of the optimal copula, methods for the choice of copulas in financial modeling are revised in Durrleman, Nikeghbali and Roncalli (2000).

Table 2

Estimated parameters for GARCH processes. Weekly data

	Peru	UK	USA
m_0	0.0051 (5.270)	0.0013 (2.325)	0.0020 (3.643)
m_1	0.1950 (5.515)	-.-	-0.1343 (4.082)
w	0.0001 (1.017)	1.4×10^{-5} (3.097)	1.1×10^{-5} (2.203)
a	0.1401 (1.519)	0.0002 (0.018)	0.0244 (1.566)
b	0.8109 (7.348)	0.8908 (38.40)	0.8769 (26.277)
g	0.0634 (1.577)	0.1594 (4.788)	0.1496 (2.879)
n	4.5679 (7.318)	10.653 (3.270)	12.849 (3.157)
l	0.1203 (2.727)	-0.1192 (-2.500)	-0.1750 (-3.849)
Log L	-2064.4	-2586.4	-2622.1
AIC	-4145.4	-5158.8	-5228.3
BIC	-4105.8	-5124.2	-5188.7

Source: Own elaboration. Note: *t* values in brackets.

The DCC was introduced by Engle (2002) as a generalization of Bollerslev (1990) Constant Conditional Correlation (CCC), the model DCC can summarize in the following equations:

$$\begin{aligned} r_{i,t} &= \sqrt{h_{i,t}\,\varepsilon_{i,t}} \qquad i=1,2; \\ h_{i,t} &= E_{t-1}\left(r_{i,t}^2\right) \\ \varepsilon_{i,t} &\sim N(0,1) \end{aligned} \tag{4}$$

Determined the returns distributions, we are able to apply the correlation definition, obtaining:

$$\rho_{12,t} = \frac{E_{t-1}\left(\varepsilon_{1,t}\varepsilon_{2,t}\right)}{\sqrt{E_{t-1}\left(\varepsilon_{1,t}^2\right)E_{t-1}\left(\varepsilon_{2,t}^2\right)}} = E_{t-1}\left(\varepsilon_{1,t}\varepsilon_{2,t}\right) \quad (5)$$

Equation (5) stated that the conditional correlation is also the conditional covariance between the standardized errors; this fact permits to estimate the dynamic correlation, which is shown in Figure 2. It could be demonstrated that, if the unconditional correlation coefficient is ρ, the conditional is given by the dynamic shown in equation (6):

$$\rho_t = (1-\alpha-\beta)\rho + \alpha\varepsilon_{t-1}^2 + \beta\rho_{t-1} \quad (6)$$

With the dynamic established in equation (6), it can be estimated the copula and correlations coefficients simultaneously; Table 3 show the copulas estimated for the studied returns.

Table 3

Copula parameters, estimated. Weekly data

	t	*t*-DCC	GDCC
n	10.439 (5.313)	15.563 (3.612)	-.-
a	-.-	0.0181 (4.583)	0.0177 (4.636)
b	-.-	0.9813 (188.5)	0.9818 (195.5)
Log L	-359.6	-414.7	-407.9
AIC	-717.2	-823.5	-811.9
BIC	-712.3	-808.6	-802.0

Source: Own elaboration. Note: *t* values in brackets.

As we can see, the correlation coefficients have grown in the last years; supported in this evidence, it is clear that the policy recommendation goes in the direction of reducing the levels of foreign investment.

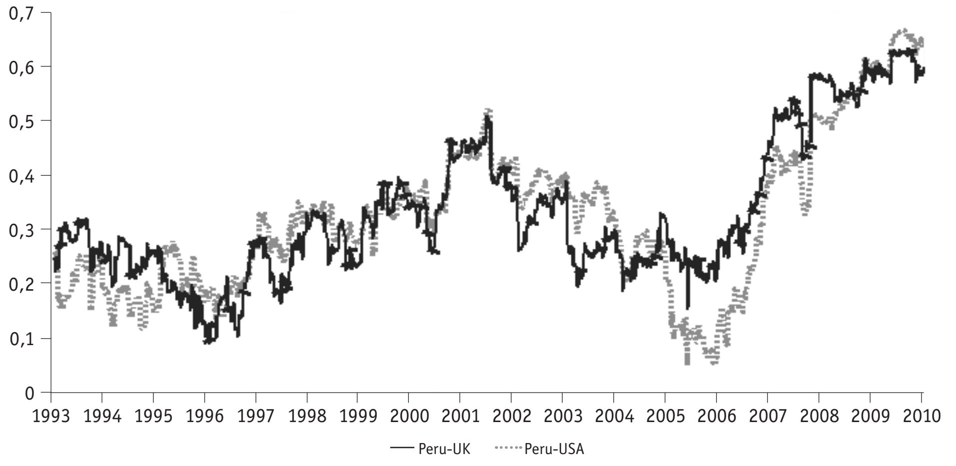

Figure 2. *t*-Dynamical Correlation Coefficients, Peru with UK and USA. Weekly data.

Source: Own elaboration.

3. The extreme events and the stocks markets

Many financial market meltdowns, originated in a specific country, have crossed the borders and triggered a hectic period in global markets. Rigobon (1999), Forbes and Rigobon (2000, 2002), Boyer, Gibson and Loretan (1999), Loretan and English (2000) claim that correlation coefficients are the same, in hectic and tranquil periods; naming the situation in which the correlation is high in all states of the world as "interdependence". Coles, Hefferman and Tawn (1999) and Poon, Rockinger and Tawn (2004) remark that under this methodology, when the correlation coefficient is not perfect ($\rho \neq 1$), the likelihood for an extreme event that occurs in a country –when the source is foreign market– is zero. The first who noticed this was Sibuya (1960). Poon *et al.* (2004) stated that the standard coefficient correlation "might lead to a significant underestimation of the risk from joint extreme events", Costinot, Roncalli and Teiletche (2000) revise some of the limits or "myths" about the correlation coefficient.

Given the evidence shown, it would be more appropriate to use an alternative definition of contagion; which can be defined "as the fact that the occurrence of a currency or financial crisis somewhere in the world increase the probability of a crisis in another country, independently of the latter's local economic and fundamental situation" (Costinot *et al.*, 2000). The contagion, from the market 1 to the market 2, is defined as:

$$P\left(X_2 > x_2 \middle| X_1 > x_1\right) > P\left(X_2 > x_2\right) \tag{7}$$

Thus, an appropriate way for measure the possibility that an extreme event generates a meltdown, in foreign stock market, is to measure it through of conditional probability: if the conditional probability is greater than the unconditional, then exists contagion; this definition has broader uses, Gençay and Selçuk (2004) use it to estimate the VaR in emerging markets, Poon *et al.* (2004) for portfolio risk, Chan-Lau *et al.* (2004) for measure contagion, Costinot *et al.* (2000) for measure contagion with extremal copulas, but (as we see later) this definition should be use in proper context because could induce to shortcomings.

Rare or extreme events, by definition, appear in the tails of the distributions; in this context, the extreme value theory is the adequate statistical tool for analyzing the realization of extreme turmoils in financial returns. The latter definition of contagion requires knowing the threshold values for which the large realizations (or exceedances) will be considered extreme value. In bivariate (or multivariate) distributions the threshold value is determined by the marginal distribution. The rare o extreme events influence the magnitude of all (distributions') moments. Mandelbrot (1963) said that "I propose to explain the erratic behavior of sample moments by assuming that the population moments are infinite, (...) Let us indeed assume that the increment $L(t,1) = \log_e Z(t+1) - \log_e Z(t)$ is a random variable with infinite population moments beyond the first". Gençay and Selçuk (2004) found that, on one side, the positive stock return in Brazil may not have a finite second moment; and, on the other, the positive stock return distribution in Argentina and Mexico have finite first, second and third moments, but may not have a finite fourth moment.[6]

Equation (7) gives a definition of the extreme events, but is necessary an operative indicator; which is given by equation (8). This definition is used by authors; we will implement the proposal by Poon et al (2004), which is a measure for bivariate distribution. If S and T are distributions in common scale, the nonparametric measure is given by:

$$\begin{aligned} X &= \lim_{q\to 1} P(q) \\ X &= \lim_{S\to\infty} \frac{P(T>s,S>s)}{P(S>s)} \end{aligned} \tag{8}$$

[6] See Table 3, Gençay and Selçuk (2004). As we see later, the existence of moments is determined by the parameter ξ of Generalized Pareto Distribution.

S and *T* are asymptotically dependent if $\chi > 0$; that is, the extreme values in each variable can occur simultaneously if they are asymptotically dependent. This means that, when infinitely large realizations always occur simultaneously, the variables are asymptotically dependent; in the same sense, the variables are asymptotically independent if the previous statement is negated. At finite levels different degrees of dependence may be attainable, because large but finite realizations of simultaneous extreme values are possible or not preclude in the asymptotically independence case.

Coles *et al.* (1999) study the asymptotically independent distributions in extreme multivariate modeling, for which $\chi = 0$ by definition; this measure can not provide information about the relative dependence for this class of models. The measure χ is inappropriate for asymptotically independent because, by definition, $\chi = 0$; as it was mentioned earlier, its use could induce shortcomings. Traditional multivariate extreme theory assumes that $\chi > 0$ when the independence of the variables is rejected (assuming the asymptotic dependence), this leads traditional methods to over estimate the likelihood of extreme events if the true value of χ is zero. To overcome these problems, Coles *et al.* (1999, p. 347) propose a new dependence measure $\bar{X}$ here the version of Poon *et al.* (2004) is shown:

$$\bar{X} = \lim_{s \to \infty} \frac{2 \log P(S > s)}{\log P(S > s, T > s)} - 1 \tag{9}$$

$\bar{X}$ is a measure of the rate at which $P(T > s \mid S > s)$ approaches zero; where $-1 < \bar{X} \leq 1$. For perfect dependence the value of this measure is one, for independence is zero. Thus $\bar{X} > 0$, $\bar{X} = 0$ and $\bar{X} < 0$ indicate that the variables are positively relationship, independent and negatively associated in the extremes of the bivariate distribution, respectively.

Coles *et al.* (1999) state that the pair $(X, \bar{X})$ is a summary of the extremal dependence; $\chi \in [0,1]$ and χ in the interval (0,1] indicates asymptotic dependence; $\bar{X} \in [-1,1]$ and if falls in the interval [-1,1) indicates asymptotic independence. Thus, $(X > 0, \bar{X} = 1)$ signifies asymptotic dependence, with the degree of dependence given by χ; $(X = 0, \bar{X} < 1)$ signifies asymptotic independence, in which case the strength of dependence is given by $\bar{X}$ Then, the recommendation is, in the first step, test the null of $\bar{X} = 1$ and, if it is accepted, χ gives the asymptotic dependence; but if it is rejected, exist asymptotic independence and $\bar{X}$ gives the strength of dependence.

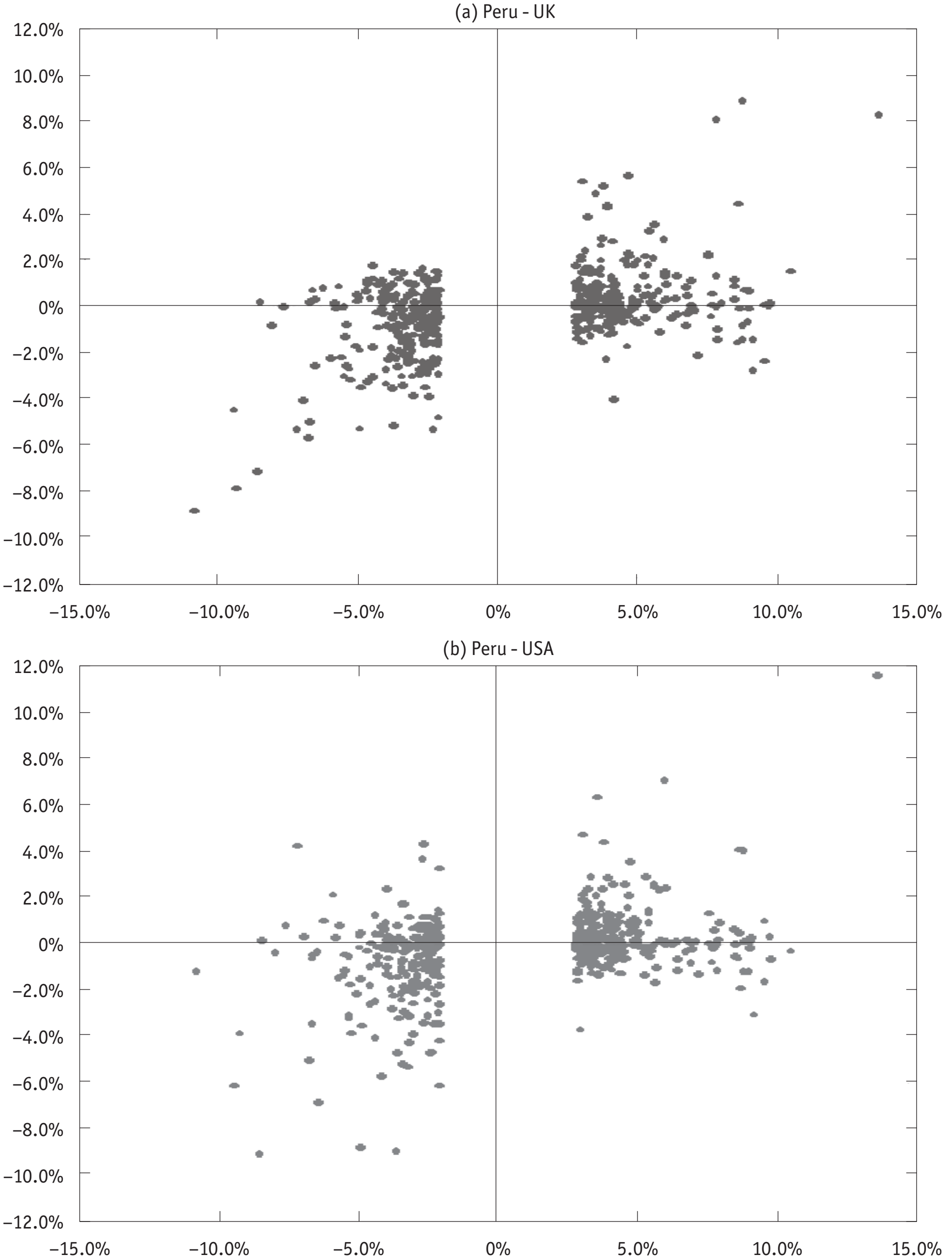

Figure 3. Highest and Lowest return of Peru with (a) UK and (b) USA.

Source: Own elaboration.

Figure 3 scatter the 250 highest and lowest returns of Peruvian stock market, in (a) with the UK returns in the same days, in (b) with the USA returns in the same days. The highest returns, in the sample, appear in the same day in the three stock markets; with the lowest the situation was similar. In the 250 lowest Peruvian stock returns, there are 169 observations of UK negative returns; while the USA negative returns added 154 observations. On the other hand, in the 250 highest Peruvian stock markets returns, there are 145 observations of UK positive returns; while the USA positive returns are 152.

Gençay and Selçuk (2004, p. 290) stated there are two approaches to study the extremal events; the first is the modelling of exceedances over a particular threshold, Embrechts, Klüppelberg and Mikosch (1997) state that the name suggested by hydrologists is Peak Over the Threshold (POT); the second approach is the Componentwise Block Maxima, which involves modeling the distribution of minimum or maximum realizations.[7] There are several studies about which approach implement. Embrechts *et al.* (1997) claim that should be use the block maxima and the GEV[8] that is, the equation (10); but McNeil and Frey (2000) use both the threshold method and the GPD[9] or the equation (11), for an AR(1)-GARCH(1,1) process. The estimation of parameters follows McNeil and Frey (2000) suggestion, these authors stated that "the application of these methods is facilitated by the (approximate) independence over the time of the residuals":

$$H_{\xi,\sigma,\mu}(x) = \exp\left[-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-1/\xi}\right] \tag{10}$$

$$G_{\xi,\sigma,\mu}(x) = 1-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-1/\xi} \tag{11}$$

The Fisher and Tippett (1928) theorem stated that the asymptotic distribution of the maxima converges to one of three GEV (named, Gumbel, Fréchet or Weibull) without considerations of the original distribution. Both Balkema and de Haan (1974) and Pickands (1975) stated that the distribution of excesses may be approximated by the GPD by

7 Coles *et al*. (1999) study the performance of the new measures in these approaches.

8 The equation (10) assumes $1 + \xi > 0$, $\xi \neq 0$ and $x > 0$.

9 The equation (11) assumes $\xi > 0$ and $x > 0$.

choosing *x* and setting a high threshold. There is a relationship between GDP and GEV, if $\log H_x(x) > -1$, $G_x(x) = 1 + \log H_x(x)$.

The parameters ξ estimated for the three countries are all positives; ξ positive indicate heavy-tail distribution (e.g. Pareto, Cauchy, etc.), ξ = 0 implicate thin-tail distribution (e.g. normal, exponential, lognormal, gamma, etc.) and x negative indicate short-tailed distributions, which have finite end-point like both uniform and beta distributions[10].

With the parameters in Table 4, for lower and upper tails of the three countries, the expected return is estimated; Table 5 shows the maximum losses and benefits in bear and bull markets for probabilities 0.1% and 99.9% with confidence interval of 95%. The maximum daily profit expected in four years is up to 12.7% for Peru, 8.0% for UK and 8.3% for USA, but the losses could be up to 9.5%, 7.6% y 8.1% for Peru, UK and USA, respectively. The greater parameter is the positive tail of UK, the 0.265 value indicate that their profits distribution could not have a fourth moment; the following is the lower-tail for the USA (0.245), also indicate that the fourth moment could not exist.

In Figure 4, the probability (0.1%) implicate that this extreme event is expected, at least, one time in every four years; that is, in each four years at least the minimum loss will go up to 9.5% in the Peruvian Stock Market, but more extreme events could emerge. As it can be seen in Figure 4, at 0.1% probability, there are losses greater than 9.5%; this implicates that there are events which appear at least once in a longer time-frequency. With 0.05% of probability and 95% of confidence, the daily losses in Peruvian Stocks markets could go up to 11.6%; that is, at least once every eight years, the stock market will go down up to 11.6%. These results could seem low when compared with other emerging markets (see Gençay and Selçuk, 2004, Table 4); this fact could be explained by the limits to daily variation in the market index. The "Director de Rueda" (Trading Director) can suspend the day-trading if the variation exceeds 7% (temporally) or steps up to 10% (permanently).[11]

---

10 For a definition of heavy-tail distribution see Gençay and Selçuk (2004).

11 This faculty is given by legal regulation, Resolución Conasev N° 095-2010-EF/94.01.1 enacted in October 4th, 2010. The last time, before concluded this paper, was June 6th, 2011.

Table 4

Parameters estimated for GPD

	Peru	UK	USA
Upper Tail			
ξ	0.1822 (3.526)	0.2649 (2.067)	0.2148 (1.876)
σ	0.0111 (15.51)	0.0072 (6.432)	0.0088 (6.880)
Threshold	1.50%	2.23%	2.23%
Quantile	0.8607	0.9743	0.9745
Exceedances	650	120	119
Log L	2157.2	439.97	419.13
Lower Tail			
ξ	0.1415 (3.076)	0.2082 (1.555)	0.2449 (2.105)
σ	0.0093 (16.61)	0.0078 (6.061)	0.0076 (6.830)
Threshold	1.18%	2.50%	2.39%
Quantile	0.8607	0.9779	0.9745
Exceedances	650	103	119
Log L	2296.8	376.07	432.07

Note: *t* values in brackets. Esceedances are values upper threshold.

Table 5

Estimated daily return at 0.1% and 99.9% with 95% of confident interval (C.I.).

	Lower Tail			Upper Tail		
	C.I. Lower	Return	C.I. Upper	C.I. Lower	Return	C.I. Upper
Peru	-0.0948	-0.0783	-0.0463	0.0525	0.1020	0.1271
UK	-0.0764	-0.0587	-0.0509	0.0506	0.0595	0.0800
USA	-0.0806	-0.0616	-0.0530	0.0542	0.0633	0.0827

Source: Own elaboration. Note: "Return" is estimated expected return; $P(x > X) = 99.9\%$ and $P(x < X) = 0.1\%$. C.I. Lower is lowest value in interval. C.I. Upper is greatest value in interval.

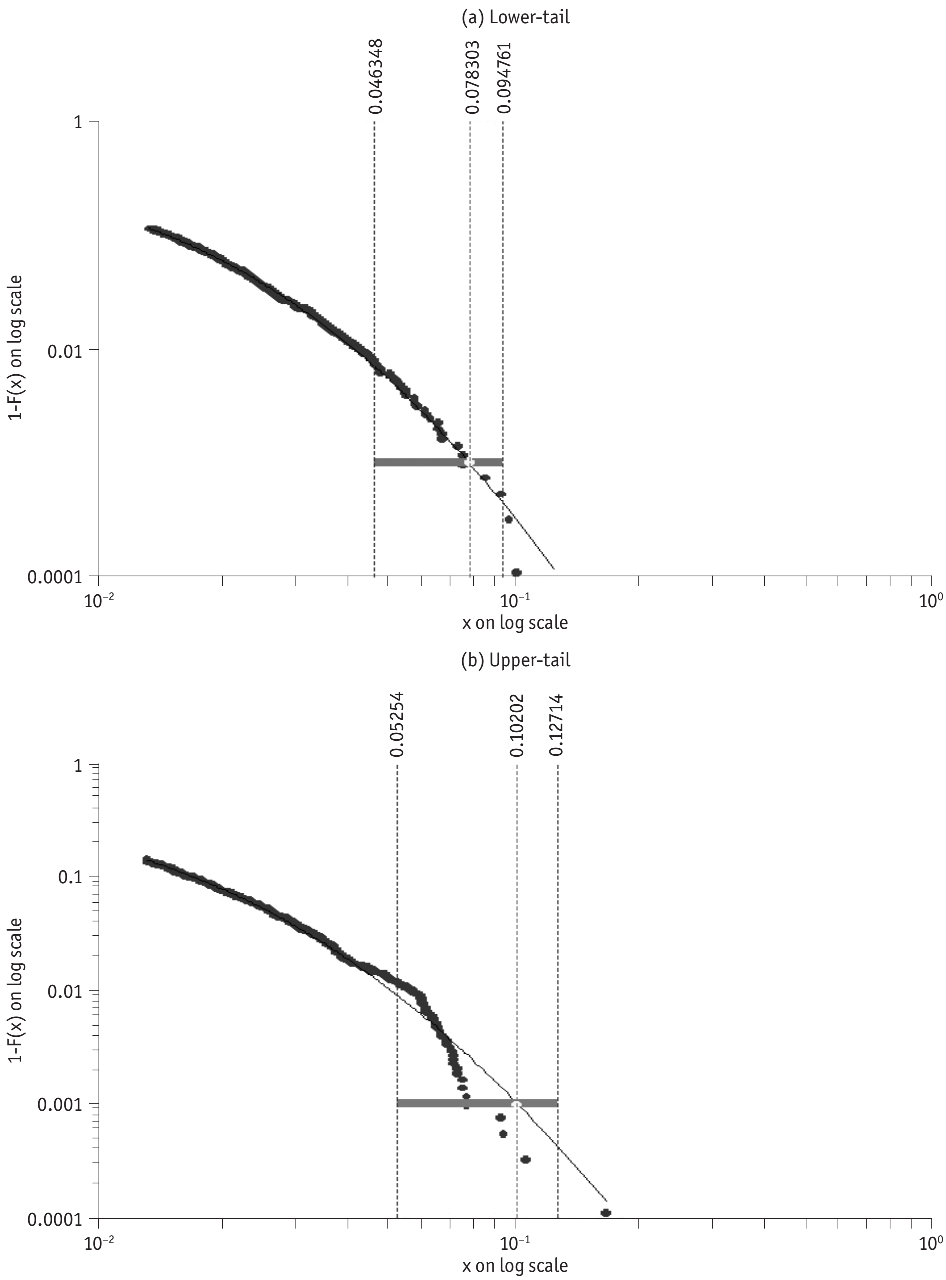

Figure 4. Extreme events at (a) Lower and (b) Upper-tail, with tails of 0.1%.

Source: Own elaboration.

The measure of dependence for bivariate random variables (*X*,*Y*) propose by Coles *et al.* (1999) assumes that the marginal distribution of X and Y are identical; in order to get identically distributed marginals, the returns distributions will be transformed to unit Fréchet marginals (*S*,*T*) as follows[12]:

$$\begin{aligned} S &= -1/\log F_X(X) \\ T &= -1/\log F_Y(Y) \end{aligned} \tag{12}$$

If $Z = \min(S,T)$, the tail index of univariate variable *Z* is the measure of dependence propose by Coles *et al.* (1999), which can be estimated by:

$$\hat{\hat{X}} = \frac{2}{n_u}\left(\sum_{j=1}^{n_u}\log\left(\frac{z_{(j)}}{u}\right)\right)-1 \tag{13}$$

With Figure 5 is determined n_u (and threshold u), the criteria is the same to choice the Hill estimator. The results are shown in Table 6; in the "Upper" row are the measures for positive tails, in the "Lower" row are the measures for negative tails, standard deviation are in brackets. As it can be appreciated, Peru is asymptotically independent of both UK and USA. The extremal dependence measure introduced by Coles *et al.* (1999) is shown in Table 6. For the upper-tails these measures are 0.66 and 0.71 for dependence of Peru with UK and USA; for the lower-tails, the values calculated are 0.83 for both UK and USA. It should be noticed that the lower-tails are greater than the upper-tails, that is, the probability the contagion in a bear market is greater than the contagion in a bull market (Coles *et al.*, 1999).

12 For a detailed explication see Poon *et al.* (2004), alternatively Coles *et al.* (1999).

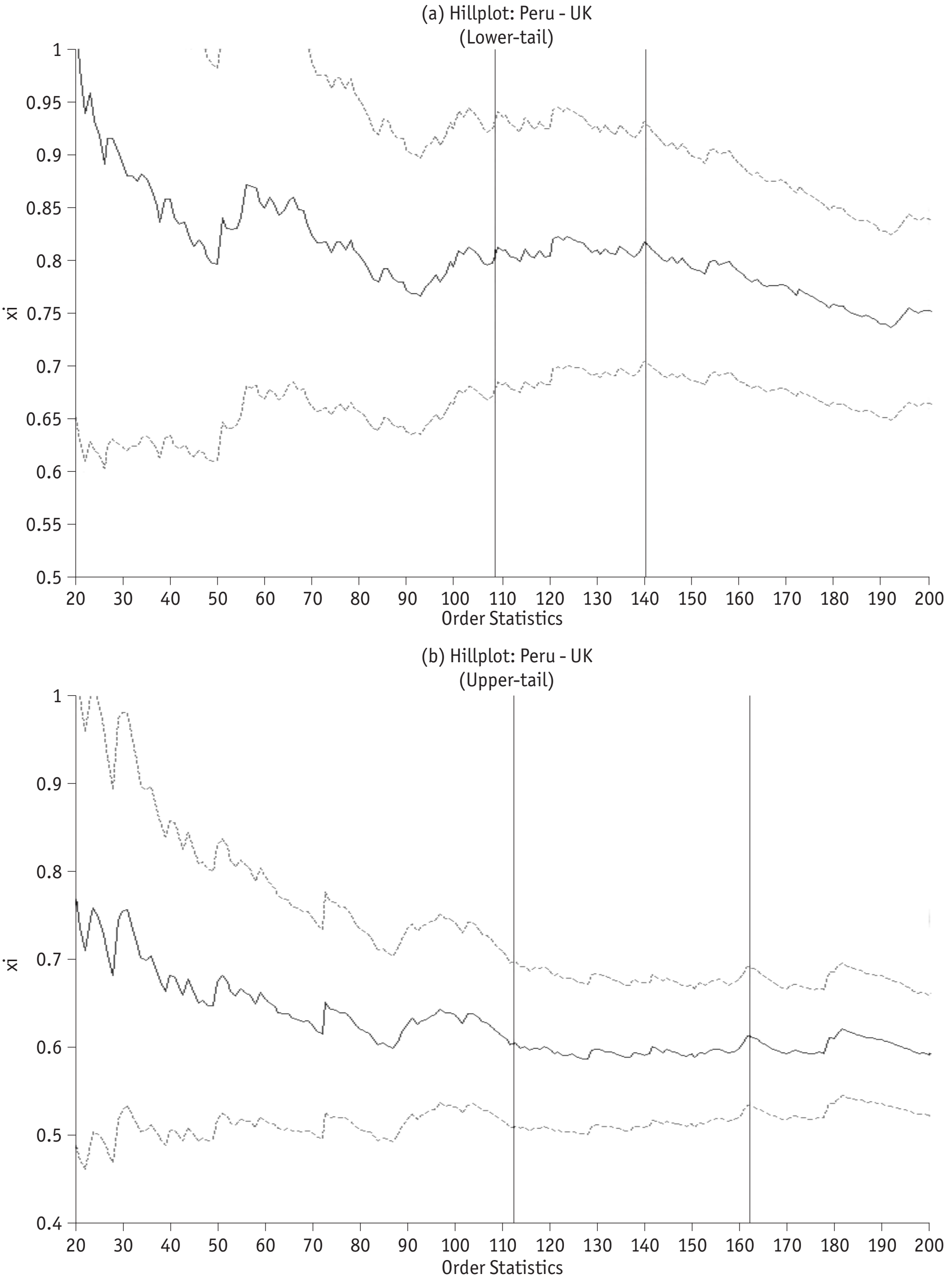

Figure 5. Determination of n_u for extremal dependence of Peru and UK.

Source: Own elaboration.

Table 6

Measures of extremal tail dependence Peru with UK and USA

Tail	Country	
	UK	USA
Upper	0.6612 (0.024)	0.7091 (0.037)
Lower	0.8290 (0.020)	0.8319 (0.067)

Source: Own elaboration. Note: "UK" are dependence parameter between Peru and UK. "USA" are dependence parameter between Peru and USA.

4. The evaluation

The Superintendent-Adjoint stated, November 16th, 2010, that: "The government discussed the possibility the foreign investment limit be of 80 percent, the same value applied in Chile, but ultimately opted for a 50 percent one"; certainly, one of the six funds could invest up to 80% in equity, such as the Peruvian Fund type-III ("Capital Appreciation"), but Antolín and Tapia (2010) show that Chilean Pensions Funds invest less than 20% in the stock market. Still more, equities represent almost 40% of total pension fund value in Peru.

Pfau (2008) evaluates the pension funds in emerging markets; in his Table 4, the author shows the optimal portfolio weights were, for Peruvian Pension Funds, the stock market share is 40% and the fixed income is 60%. For the most profitable countries in Latin American, Mexico (25.7%) and Colombia (22.8%), the weights of stock markets are 41.1% and 27.7% for Mexico and Colombia, respectively; Pfau (2008) suggests that Chilean Pension Funds should invest up to 24.8% of pension fund value in equity markets and only these funds could invest in foreign stock markets (up to 11.4% of total pension fund value).

Figure 6 shows the evolution of interest rates: CD-BCRP, T-Bill and Libor; these rates can be seen as proxies of fixed-income return. Table 7 shows the return of Pension Funds in four countries, in the period 2008-2010.

Figure 6 shows the return for CD issued by Central Reserve Bank of Peru (BCRP), daily evolution of annualized rates in domestic currency (New Soles); T-Bill is the 4-week Treasury Bill secondary market rate. The return in domestic currency is higher than foreign fixed-income return.

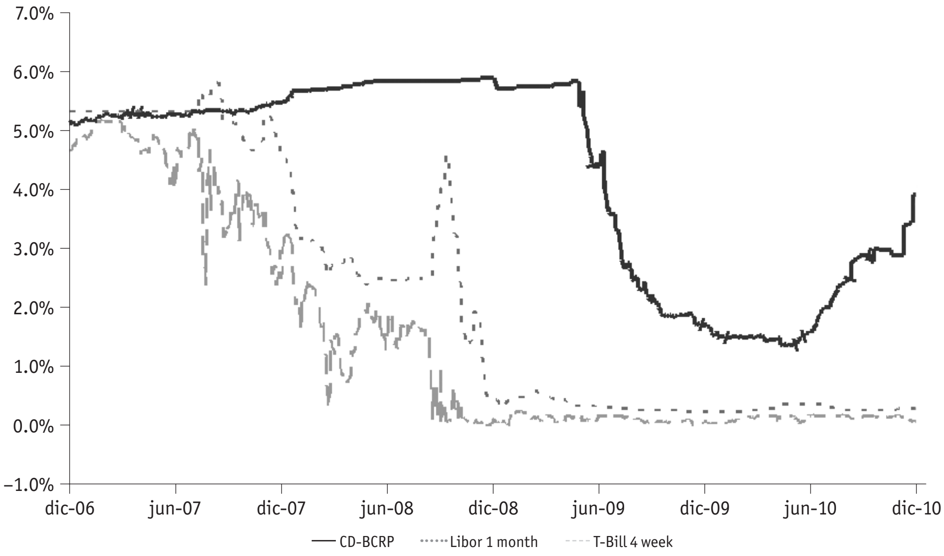

Figure 6. Interest rates: CD-BCRP, Libor and T-Bill.

Source: Own elaboration.

Table 7

Annual Return Percentage in domestic currency

		2008	2009	2010
Peru	Annual	-26.7	32.9	16.0
	Accumulated	-26.7	-2.6	13.0
Chile	Annual	-22.0	27.7	9.3
	Accumulated	-22.0	-0.4	8.9
Dominican Republic	Annual	7.2	7.8	4.3
	Accumulated	7.2	15.6	20.5
Colombia	Annual	0.2	11.4	15.6
	Accumulated	0.2	11.2	28.5

Source: Own elaboration. Note: Source is FIAP.

Table 7 shows the annual and accumulated return for period 2008-2010. The highest annual return is for Peruvian Pension Funds (32.9%), but it has also the lowest (-26.7%). The highest accumulated return is for Colombia (28.5%) and the lowest is for Chile (8.9%). In 2008, Colombia had 72.1% in fixed-income and 25.8% in equities; in 2010 the portfolio had another composition, fixed-income represented 49.9% of total portfolio value and equities 45.1%. In 2008, Peru applied 60.3% of funds to fixed-income and 40.5% to equities; in year 2010, 46.8% was applied to fixed-income and 53.2% to equities. Pension Funds in Dominican Republic only can invest in fixed-income.

As it can be seen, the profitability was not dependent on higher levels of investment in stock markets: The return accumulated in Peruvian Pension Funds was 13.0%, with over 40% in equities; while Dominican Pension Funds got 20.5%, with 100% in fixed-income. Colombian Pension Funds used an optimal mixed and got 28.5% as return accumulated in the analyzed term.

The picture is not complete, the statistics show the returns but not the risk level, although it gives an idea about the optimal investment policy. A complete analysis requires the inclusion of the risk levels associated to returns got by the portfolio investment.[13] Another important aspect is the levels in foreign investment, in Peru it was 12.4% in 2008 and stepped up to 26.4% in 2010. In Dominican Republic was null in the three years. Chilean Pension Funds applied 28.5% for foreign investment in 2008 and 45.3% in 2010. Colombia allocated 9.4% and 12.6% in foreign investment in 2008 and 2010, respectively.

The moral of the fable is that the Peruvian Pension Funds were moved as a "*montaña rusa*" (roller coaster); the profitability is dependent on when you get in and when you get out, when you are up or when you are down.

5. Conclusion

In this paper, the supervisor's proposal to step up the foreign investment bounds is analyzed. Two aspects are measured: The first one is the markets dependence, measured through the correlation coefficients while the second is the contagion of extreme events. For the latter the extreme value theory is applied.

The investment in foreign stock markets is evaluated through the evolution of Dynamical Conditional Correlation (DCC) introduced by Engle (2002). This indicator shows that the correlation (and integration) of the markets has increased in recent times; the results implicate that the diversification has lost effectiveness. Also the joint distribution was

13 The sample standard deviation in the three years was 30.7% for Peru, 25.1% for Chile, 1.9% for Dominican Republic and 8.0% for Colombia.

estimated, for this objective copulas were used; if the joint distribution is an elliptical, the correlation coefficient gives the dependence between the two variables. In the first step the marginal distribution was estimated, the series present volatility clustering or ARCH/GARCH processes; in the next step we estimated the joint distribution, the copulas permit us to get the parameter of dependence between two variables without pre-specifying the exacts marginals distributions and dependence between the variables. Normally, with the specification of the joint distribution; both the marginal distributions and the dependence were obtained; with the copulas, we can model the marginals distributions and, in the next step, get the dependence parameter and the joint distribution.

Another measurement used to evaluate the contagion between countries is the asymptotic dependence; Coles *et al.* (1999) proposed to use $(X, \bar{X})$ as measures of dependence. The extreme value theory analysis only uses the extreme values; the Peak Over the Threshold (POT) permits analyzing the extremes both upper and lower tails independently, the results show that the contagion of negative shocks are stronger than positive shocks.

With this evidence, the main policy recommendation is to reduce the levels of equities and foreign investment in the portfolio of Peruvian Pension Funds.

The analysis went ahead and evaluated the term 2008-2010. In a reduced sample, the countries without foreign investment had better performance than countries with foreign investment; the countries with high participation of equities had a performance worse than the one of countries with middle and null investment in equities, that is, with high o full investment in fixed-income. For Peruvian Pension Funds, the moral is “the profitability is dependent on when you get in and when you get out, when you are up or when you are down”; the evolution of returns is, hence, a roller coaster.

The implied conclusion seems amazing: lower weights in both equities and foreign investment come with higher returns; however, it is not so much of a surprise. Pereda (2007), Campodónico (2008a,b), Ortiz *et al.* (2010) and Chirinos and Ortiz (2013) got similar conclusions.

This study can be extended in a couple of directions. The bounds for both shares and foreign investment have not been estimated; one problem may emerge: the risk-free interest rate in foreign currency is not risk-free rate in domestic currency. This fact could be a problem when the weights of foreign investment in the portfolio are evaluated; the premium for risk is not the same for both domestic and foreign investors. The solution to this problem will permit us to get the weights of the optimal portfolio.

There is evidence that fixed-income is more relevant than variable-rent, but the domestic currency rates of bonds are unpredictable; this fact could difficult the allocation between different types of bonds. The fixed-income domestic rates are greater than foreign interest rates, even if the tendency of domestic currency appreciation is considered; these two facts support the recommendation of reducing the investment in foreign bonds.

Finally, this paper has analyzed the optimal portfolio, a natural extension is the application to Value-at-Risk (VaR) or another one coherent measures of risk. The measures used have assumed only one period; Chan-Lau *et al.* (2004) estimated the asymptotic dependence for a sub-sample and tested if these measures changed. This paper has concentrated its interest in only two markets, but the analysis can include other stock markets too. There is evidence that Latin American countries are integrated with others countries in the region and the analyzed markets. The Peruvian Pension Funds invest in other countries with a little relative weight in the portfolio, but is necessary to know how much integrated are these others markets with the domestic stock market.

References

Agencia Andina (2010, November 16). Ampliación de límite de inversión de AFP a 50% implicará US$ 5,000 millones hacia el exterior. Retrieved from http://www.andina.com.pe/espanol/Noticia.aspx?id=OnQTwyZQlEY=

Alexander, C. (2001). *Market models: A guide to financial data analysis*. New Jersey: John Wiley and Sons Inc.

Antolín, P. and W. Tapia (2010). Investment performance of privately managed pension funds: Overview of the available data. In: Hinz, R. *et al*. *Evaluating the Financial Performance of Pension Funds* (pp. 25-37). Washington D.C.: The World Bank.

Arouri, M.; F. Jawadi and D. Nguyen (2010). *The dynamics of emerging stock markets: Empirical assessments and implications*. Berlin, Heidelberg: PhysicaVerlag.

Balkema, A. and L. de Haan (1974). Residual life time at great age. *Annals of Probability,* 2 (5), 792-804.

Black, F. (1972). Capital market equilibrium with restricted borrowing. *The Journal of Business*, 45 (3), 444-455.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, 31 (3), 307-327.

Bollerslev, T. (1990). Modeling the coherence in short-run nominal exchange rates: A multivariate generalized ARCH model. *The Review of Economics and Statistics*, 72 (3), 498-505.

Boyer, B.; M. Gibson and M. Loretan (1999). Pitfalls in tests for changes in correlations. *International Finance Discussion Papers*, 597, March.

Campbell, R.; K. Koedijk and P. Kofman (2002). Increased correlation in bear markets. *Financial Analysts Journal*, 58 (1), 87-94.

Campodónico, H. (2008a). AFPs: hay demasiada plata en la Bolsa. *La República*, Lima, Perú. Recuperado el 10/27/08 de http://www.cristaldemira.com

Campodónico, H. (2008b). Pésima química: Bolsa de Valores e inversiones de AFP. *La República*, Lima, Perú. Recuperado el 10/11/08 de http://www.cristaldemira.com

Chan-Lau, J., D. Mathieson and J. Yao (2004). Extreme Contagion in Equity Markets. *IMF Staff Papers*, 51 (2), 386-408.

Chirinos, M. (2013). Medición de contagio e interdependencia financieros mediante cópulas y eventos extremos en los países de América Latina. *El Trimestre Económico*, LXXX (1), 169-206.

Chirinos, M. and D. Ortiz (2013). Evaluación a la modificación de límites de inversión: El desempeño de las AFP en el Perú 20052009. En: Vera, M. y Z. Melgarejo (Comps.), *Investigación global en contabilidad y finanzas* (pp. 456-476). Bogotá: CID, Universidad Nacional de Colombia.

Coles, S.; J. Hefferman and J. Tawn (1999). Dependence measures for extreme value Analysis. *Extremes*, 2 (4), 330-365.

Costinot, A.; T. Roncalli and J. Teiletche (2000). Revisiting the dependence between financial markets with copulas, Working Paper. Retrieve from: http://www.thierry-roncalli.com

Durrleman, V.; A. Nikeghbali and T. Roncalli (2002). Which copula is the right one? Working paper. Groupe de recherche opérationnelle crédit Lyonnais. Retrieve from: http://www.thierry-roncalli.com

Embrechts, P.; C. Klüppelberg and T. Mikosch (1997). Modelling extremal events: for insurance and finance. Berlin Heidelberg New York: Springer-Verlag.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 50 (4), 987-1007.

Engle, R. F. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business* and *Economic Statistics*, 20 (3), 329-350. DOI:10.1198/073500102288618487.

Fisher, R. A., and L. Tippett (1928). Limiting forms of the frequency distribution of the largest or smallest member of a sample. *Proceeding of Cambridge Philosophical Society*, 24, 180-190. Retrieve from: http://digital.library.adelaide.edu.au/dspace/handle/2440

Forbes, K. and R. Rigobon (2000). Contagion in Latin America: Definitions, measurement, and policy implications. NBER: Working Paper 7885.

Forbes, K. and R. Rigobon (2002). No contagion, only interdependence: Measuring stock market comovements. *The Journal of Finance*, 57 (5), 2223-2261.

Gençay, R. and F. Selçuk (2004). Extreme value theory and value-at-risk: Relative performance in emerging markets. *International Journal of Forecasting*, 20 (2), 287-303.

Glosten, L.; R. Jagannathan and D. Runkle (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *Journal of Finance*, 48 (1), 779-801.

Grinblatt, M. and S. Titman (1993). Performance measurement without benchmarks: An examination of mutual fund returns. *The Journal of Business*, 66 (1), 47-68.

Gupta, R. and T. Jithendranathan (2008). Time-varying correlations and optimal allocation in emerging market equities for Australian investors: A study using East European depositary receipts. *International Research Journal of Finance and Economics*, 18, 127-141.

Hansen, B. (1994). Autoregressive conditional density estimation. *International Economic Review*, 35 (3), 705-730.

Jondeau, E. and M. Rockinger (2006). The Copula-GARCH Model of conditional dependencies: An international stock market application. *Journal of International Money and Finance*, 25 (5), 827-853.

Loretan, M. and W. English (2000). Evaluating "correlation breakdowns" during periods of market volatility. *International Finance Discussion Papers*, 658, February.

Mandelbrot, B. (1963). The variation of certain speculative prices. *The Journal of Business*, 36 (4), 394-419.

McNeil, A. and R. Frey (2000). *Estimation of Tail-Related Risk Measures for Heteroscedastic Financial Time Series: An Extreme Value Approach*. Journal of Empirical Finance, 7, 271-300.

Ortiz, D.; M. Chirinos and Y. Hurtado (2010). La frontera eficiente y los límites de inversión para las AFP: una nueva mirada. *Journal of Economics, Finance and Administrative Science*, 15 (29), 95-117.

Patton, A. (2006). Modelling Asymmetric Exchange Rate Dependence. *International Economic Review*, 47 (2), 527-556.

Pereda, J. (2007). Estimación de la frontera eficiente para las AFP en el Perú y el impacto de los límites de inversión: 1995-2004. Serie Documentos de Trabajo N° 2007-009. Retrieve from: http://econpapers.repec.org/paper/rbpwpaper/2007-009.htm

Pfau, W. (2008). Emerging market pension funds and international diversification. GRIPS Policy Information Center. Discussion Paper 08-2010.

Pickands, James, III (1975). Statistical inference using extreme order statistics. *Annals of Statistics*, 3 (1), 119-131.

Poon, S-H; M. Rockinger and J. Tawn (2004). Extreme value dependence in financial markets: Diagnostics, models, and financial implications. *The Review of Financial Studies*, 17 (2), 581-610.

Rigobon, R. (1999). On the measurement of the international propagation of shocks. NBER: Working Paper 7354.

Rivas-Llosa, R. and G. Camargo (2002). Límites de inversión para las AFP: ¿son financieramente eficientes? *Punto de Equilibrio*, 79, 40-41.

Sibuya, M. (1960). Bivariate extreme statistics, I. *Annals of the Institute of Statistical Mathematics*, 11 (2), 195-210.

Tse, Y. and A. Tsui (2002). A multivariate generalized autoregressive conditional heteroscedasticity model with time-varying correlations. *Journal of Business* and *Economic Statistics*, 20 (3), 351-362.

Vogiatzoglou, M. (2010). *Dynamic Copula Toolbox*. Working Paper. Retrieve from: www.mathworks.com.