A Methodological Approach for the Valuation of Callable Bonds in Emerging Markets: The TGI Example*

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* This paper is one of the results of finance research projects financed by CESA and PUJ. This paper is a detailed and refined example of how to value callable bonds in emerging markets using real market data. This example is based in great part in the methodological approach developed by Salomon Brothers as explained in Fernando Rubio, *Valuation of Callable Bonds: The Salomon Brothers Approach* (July 2005). The article was received on 13-05-2008 and was accepted for publication on 28-10-2009.

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Abstract

This article aims to shed light on the issues that stock brokers face upon implementing the binomial model when valuating corporate bonds with a multiple exercise option for the issuer. To that end, the proposed methodology is used to valuate this type of instrument in the company Transportadora de Gas del Interior Internacional Ltda. (TGI). In the specific case of TGI, it was found that the binomial model enables finding the value of the spread points that can be attributed to the option and that, employing that measure, the sole risk measure attributable to a specific corporate activity can be obtained.

Key words:
Valuation, callable bonds, OAS, emerging markets.

Una estrategia metodológica para la valoración de bonos con privilegio de redención anticipada en los mercados emergentes: el caso de la Transportadora de Gas del Interior Internacional

Resumen

El propósito del artículo es clarificar algunos de los problemas que los profesionales de la bolsa encuentran al implementar el modelo binomial en la valoración de bonos corporativos con opciones de ejercicio múltiple por parte del emisor. Para ello se propone una metodología que valora este tipo de instrumentos, utilizando los bonos de la Transportadora de Gas del Interior Internacional Ltda. (TGI). En el caso específico de la TGI se encontró que empleando el modelo binomial es posible hallar el valor de los puntos de *spread* atribuibles a la opción, y con esta medida también obtener una medida del riesgo único atribuible a una actividad corporativa específica.

Palabras clave:
valoración, bonos redimibles, OAS, mercados emergentes.

Uma aproximação metodológica para valoração de bônus corporativos em mercados emergentes: o exemplo da Transportadora de Gás do Interior Internacional

Resumo

O propósito do artigo é esclarecer alguns dos problemas que os profissionais da bolsa encontram ao implementar o modelo binomial na valoração de bônus corporativos com opções de exercício múltiplo por parte do Banco Central. Para isso propõe-se uma metodologia que valoriza este tipo de instrumentos, utilizando os bônus da Transportadora de Gás do Interior Internacional Ltda. (TGI). No caso específico da TGI encontrou-se que empregando o modelo binomial é possível descobrir o valor dos pontos de *spread* atribuíveis à opção, e com esta medida obter também uma medida do risco único atribuível a uma atividade corporativa específica.

Palavras chave:
valoração, bônus corporativos, OAS, mercados emergentes.

Introduction

Unlike the pricing of equities, and setting the issue of credit quality aside, the pricing of bonds depends solely on the future behavior of interest rates and their effect in discounting future expected cash flows. Where bonds have embedded calls from the issuer, this represents a distinct challenge, because the issuer can alter the nature of the cash flows that the investor will receive depending on the future behavior of interest rates.

Therefore, given the fact that the issuer can recall the bond at his convenience, the investor faces a substantial risk of prepayment from the part of the issuer. This characteristic can often be detrimental to the investor, because usually the issuer will recall the bond at a higher discount rate than that which can be obtained in the open market, thus generating a loss to the investor who is forced to sell the bond back to the issuer at a price below the real market value of the bond at the future time of the transaction (Rubio, 2005). Since the investor faces the risk of an uncertain stream of cash flows, the common market practice is to demand a higher yield in a callable bond than in a non-callable bond in order to compensate the higher risk caused by the embedded call options in a specific issue.

In common practice, the credit and liquidity risk of any common non-callable bond is determined by the additional yield spread paid by that bond when compared to the yield of a risk-free bond with a similar maturity date (i.e. Corporate Issues vs. U.S. Treasuries). In the case of callable bonds the additional spread demanded by the investor over and above the credit and liquidity risk premium is known as the Option Adjusted Spread (OAS). In order to calculate the OAS, assumptions have to be made about the behavior of the uncertainty of the stream of cash flows of the bonds and their effect on future yields, and therefore modeling risk is a factor that has to be taken into account when valuing callable bonds (Henderson, 2003). In the US numerous studies have been conducted regarding the behavior of the OAS of callable vs. non-callable bonds. For example, Longstaff (1992) found that the implicit call values in callable US treasuries are sometimes overpriced in comparison to their theoretical value due to negative option values. This claim was later contested by Edleson et al. (1993) who demonstrated that the apparent mispricing was not caused by negative option values, but by factors attributable to other risks. Dolly (2002) found that in average the call value of US corporate callable bonds during the period 1973-1994 was 2.25% of par, and that the price patterns are consistent with those one should expect from commonly-used option pricing models. In the specific case of TGI, there is an additional factor that must be taken into account: country risk.

The problems that an investor faces with sovereign risk are not easy to handle because there are a series of factors than can affect the spread attributable to this specific kind of risk. For example Eichengreen and Moody (1999) found that market sentiment was instrumental in determining emerging market spreads in 1994-1996. Also, according to Erb et al. (1999), one the greatest challenges in emerging market bond valuations is the nature of the term structure of interest rates. Given the fact that in times of crisis, returns are highly correlated with those of emerging market equities, this generates tracking errors that alter the nature of the term structure of interest rates in those markets over certain periods of time. This means that when dealing with emerging market issues, such as that used as an example in this paper, care must be taken to use models that really capture the short- and long-term volatilities that affect interest rates relevant to a given debt issue.

Finally, our specific objective is to use a practical example to show how the binomial pricing model can be used to determine the OAS and the specific risk of a callable bond issued by a company located in an emerging market by using a market-based approach when incorporating the company's country risk spread.

1. The Binomial Pricing Model: A Simple Approach for Valuing Embedded Options in Callable Bonds

According to Rubio (2005) it is preferable to use the binomial pricing model rather than the Black-Scholes model when valuing callable bonds. This is because Black and Scholes incorporate the following assumptions into the model, when most of the time they do not apply to bonds and the term structure of interest rates in general:

1. Black and Scholes assume that interest rates are constant through the life of the bond, this assumption is not realistic since all bonds have reinvestment risk, except in the case of zero-coupon bonds.

2. Black and Scholes assume an infinite log-normal price distribution which is true for stocks, but not for bonds, since the later have a known time to maturity.

3. Constant volatility through the period of valuation, which in the specific case of bonds is not just a function of price, but is a function of variability in interest rates that tend to change over time as the bond nears maturity.

The binomial model as proposed by Cox-Ross-Rubinstein (1979) is preferable to that of Black-Scholes when valuing callable bonds. The main reason for this is that even though closed-form option pricing models (i.e. Black and Scholes) are easier to handle, those models do not capture many of the features required in the valuation of a callable bond. Specifically, the Black-Scholes model is extremely inaccurate in capturing the variations of interest rates throughout the life of the option as well as the embedded value of multiple call options after the first settlement date. Although in practice, when a Binomial Model is taken to the "limit" its results tend to converge with those obtained by Black and Scholes, this occurs because the Binomial Model is simply a discrete approximation of the underlying stochastic differential equation used in Black and Scholes. Given that the Binomial Model distinctive feature is the use of discrete periods, this feature is what gives the Binomial Model a certain advantage over Black and Scholes in the specific case of valuing multiple embedded options in callable bonds. This is so because the model assumes (in the specific case of bonds) that the yield of the security evolves on step to step basis as times passes (Wong, 1993). The Binomial Pricing Model assumes that the underlying asset price or yield evolves in a multiplicative binomial pattern in the following manner:

Any node for the price of the asset (*S*) in the lattice tree should go up by an upward factor (*u*) with a probability (*P*) or by a downward factor (*d*) with a probability (*1-P*) for multiple periods in the following manner (Figure 1).

In a similar manner we value the price of the call option using a risk-neutral probability approach at each node of the lattice using the following formula[1]:

$$C_{t-1} = \frac{1}{(1+r_f)} \times (\, p \; x \; C_{tu} \; + \; (1-p) \; x \; C_{td} \,)$$

In which C_{t-1}=Call value for the preceding period

r_f =The proxy variable for the theoretical risk-free interest rate for a given period

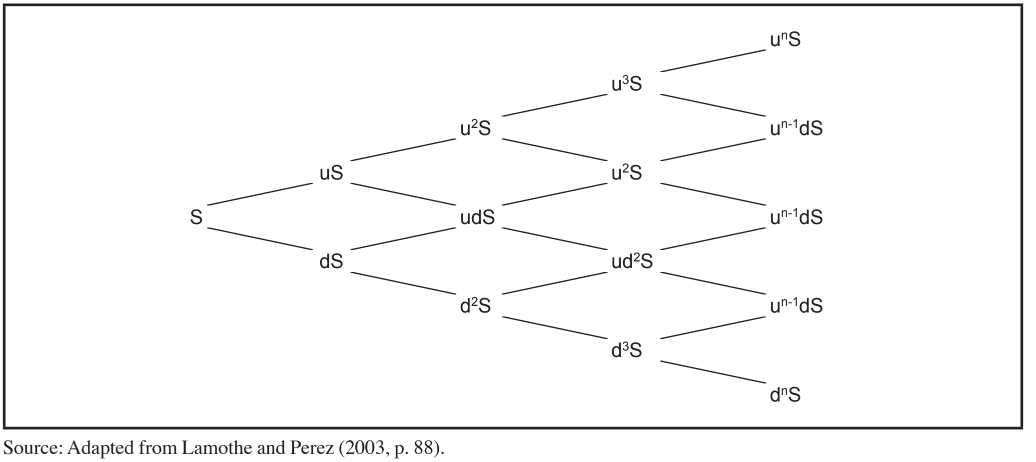
C_{tu}=The call value for the immediately posterior upward node

C_{td}=The call value for the immediately posterior downward node

$P=((1+r_f)–d)/(u–d)$ or the risk-neutral probability of an upward movement of a replicating portfolio (short or long in a call option, or long or short in risk-free bond) where (*u*) is an upward factor and (*d*) is a downward factor.

Figure 1

Binomial Price Lattice



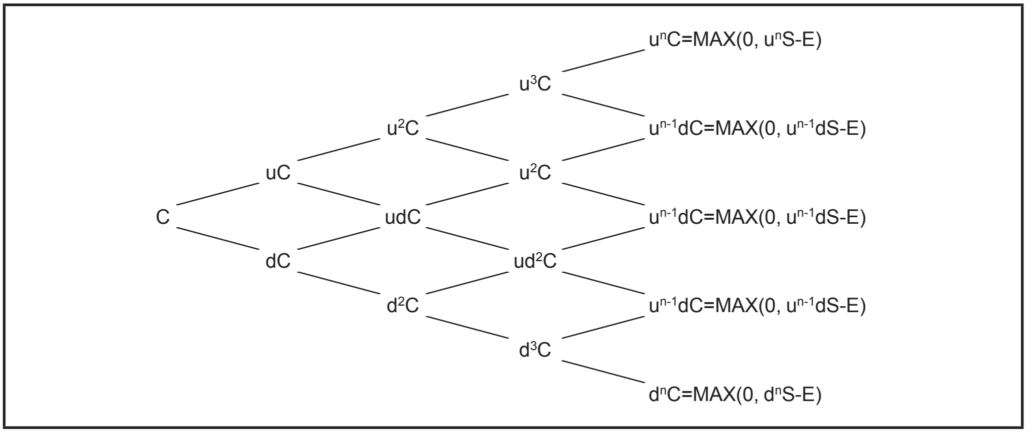
Source: Adapted from Lamothe and Perez (2003, p. 88).

In order to find the European option value at each node, the formula is applied backwards in each node of the following lattice (based on the nominal value obtained for the option of each node at its maturity) (Figure 2).

[1] For a complete development of the algebraic process necessary for finding risk-neutral probabilities and the theoretical background of the principles behind the replicating portfolio inherent in the binomial option pricing formula, we recommend the book *Opciones financieras y productos estructurados* (2003), by Prosper Lamothe Fernández and Miguel Pérez Somalo, pp. 79-90.

Figure 2

Binomial Call Price Valuation Lattice



Source: Adapted from Lamothe and Perez (2003) p. 90.

Where E is the strike price of the option being valued at a specific point in time (*t*), if any value of *S* is greater than *E* at maturity the option will be exercised, otherwise its value will be cero (0).

Therefore this approach can be used in valuing multiple embedded options, because by using a lattice we can incorporate irregular and path dependant values during the time to expiration of the option. If indeed, the option is not exercised at a specific node, this means that those cash flows will remain until the next option in the theoretical call schedule expires. By doing this in a repetitive manner, all the calls scheduled in the callable bond will be incorporated into the valuation model. In this way is possible to determine the value of each call embedded on the bond, and how the values of these calls affect the price of the bond and its expected future yield at a specific point in time.

2. A Simple Methodological Approach for Implementing the Binomial Option Pricing Model for Valuing Callable Bonds: The TGI Example[2]

The main problem faced in option valuation is how to find the appropriate proxy variables to be used as inputs of the model. Therefore, the main objective of this paper is to use a practical example on the steps required to value a callable bond using the binomial pricing model. In order to develop a meanin-

[2] Although (Ritchken, 1995) made a well-augmented point over the advantage of trinomial trees over binomial trees on the grounds that with an additional degree of freedom move spacing can be independent over move timing a trinomial tree. This advantage offers a better approximation for short term options. In the long term such differences are negligible and both models tend to converge. For more relevant information on the subject we recommend the working paper "On the Relation Between Binomial and Trinomial Option Pricing Models" written by Mark Rubinstein (2000) and that is available at the following website: http://www.haas.berkeley.edu/groups/finance/WP/rpf292.pdf.

gful example of how to develop the binomial pricing model, the example will be focused on the valuation of a recent issue by TGI International Ltd. which is a subsidiary of a Colombian company called Transportadora de Gas del Interior, a local monopoly whose business is the transportation and wholesale distribution of natural gas. The issue has the characteristics presents in Table 1 (Note: For the purpose of this example, and for the remainder of the document, the valuation date is March 31, 2008).

Table 1

TGI YTM as of March 31, 2008

Issuer	TGI INTERNATIONAL LTD
Country	Colombia
Maturity	October 3, 2017
Coupon	Fixed 9.5% Semi Annual
Day Count	30/360
Fitch Rating	BB
Yield (3/31/2008)	8.872%

Source: Bloomberg (s. f.).

The issue has four embedded call options from the issuer and its call schedule is as follows in Table 2 (it is important to remember that on any coupon payment date the clean price is equal to the dirty price).

Table 2

TGI Call Schedule

Date (mm/dd/yyyy)	Exercise Price
10/03/2012	104.750
10/03/2013	103.167
10/03/2014	101.583
10/03/2015	100.000

Source: Bloomberg (s. f.).

The following are some of the problems of how to obtain meaningful proxy variables in order to value this specific issue:

1. Finding a proxy for the risk-free rate, given the fact that even though the issue is dollar- denominated, the company in question is not US based.

2. Finding a proxy for the volatility of the yield of the proxy used as a risk-free rate that incorporates the additional spread required for country risk.

3. Finding a proxy for a non-callable bond issue with the same coupon and maturity date comparable to the issue that is being valued.

4. Finding the spread attributable to specific industry risk.

Therefore, in order to provide a meaningful insight on how to address these issues, a detailed step-by-step methodological approach is described in the process required to value TGI callable bond issue throughout this paper.

2.1 *Step 1-Colombian Sovereign Bonds Yield as a Proxy Variable that Incorporates the Additional Spread Required by Country Risk*

Before implementing the lattice approach for predicting the behavior of future yields for the specific case of TGI, it was necessary to find a proxy for a non-callable bond with the same coupon and maturity dates of TGI. Since TGI is located in an emerging market there are no comparable issues from a non-callable bond in order to determine the OAS of TGI. Therefore, in order to find a meaningful proxy for a non-callable bond a synthetic theoretical non-callable bond series was created in order to find a meaningful yield that incorporated both the risk-free rate and a spread attributable to country risk[3]. This theoretical yield was found through linear interpolation using two Colombian sovereign issues with a maturity date before and after TGI maturity date. The issues have the characteristics presents in tables 3 and 4.

Table 3

Colombia 2017 Sovereign Bond

Issuer	REPUBLIC OF COLOMBIA
Country	Colombia
Maturity	January 27, 2017
Coupon	Fixed 7.375% Semi Annual
Day Count	30/360
Fitch Rating	BB+
Yield (3/31/2008)	5.803%

Source: Bloomberg (s. f.).

Table 4

Colombia 2020 Sovereign Bond

Issuer	REPUBLIC OF COLOMBIA
Country	Colombia
Maturity	February 25, 2020
Coupon	Fixed 11.75% Semi Annual
Day Count	30/360
Fitch Rating	BB+
Yield (3/31/2008)	6.091%

Source: Bloomberg (s. f.).

Therefore, the time left to maturity for the Sovereign Bonds expressed in years[4] is 8.82222 and 11.90 respectively, also the time left to maturity for expressed in years for TGI is 9.50555. Since we know the yield to maturity and the time left to maturity of both bonds, we can use a simple interpolation formula to find the theoretical yield of a Colombian sovereign bond that pays a 9.5% fixed semiannual coupon and matures on October 3, 2017 in the following way:

$$5.867\% = 5.803\% + [(9.508333 - 8.82222) \times (6.091\% - 5.803\%)/(11.90 - 8.2222)]$$

In this way, we find that the theoretical yield for a Colombian sovereign bond with the same maturity date as TGI would be approximately 5.867%. Given that this simple approach has tremendous conceptual flaws we opted to use a more robust term structure model which for this specific case was the Nelson and Siegel model. The Nelson Siegel Model formulation gives a conservative representation of the forward rate function given by (Abad and Benito 2005):

$$r(t) = \beta_0 + \beta_1 e^{\frac{-t}{\tau}} + \beta_2 \frac{t}{\tau} e^{\frac{-t}{\tau}}$$

3 In other words, a yield that incorporates the required country risk spread over a US treasury with similar maturity.

4 To obtain the exact time from the 31 of March 2008 until the date of maturity, we first calculate the time left in a semiannual basis (S/A basis), this is done in order to take into account all the coupons left as well as the principal. Then we express the time in an annual basis, because the yields are expressed by the market in an annual basis. Also the fraction is to denote the time left from the current date until the next coupon payment. In the specific case of TGI, in a semiannual basis, this fraction is expressed as 0.0166667. That gives us in total 19.0166667 semiannual periods that divided by two gives us 9.508333 years.

Where the parameters β_0, β_1, β_2, and τ are obtained by finding the rate for a time (t) for different maturities and by maximum likelihood fitting the rate obtained by the formula to the actual observation by minimizing the MSE for each actual vs. calculated observation for the term structure for an observable time period for which our specific case was one year. For calculating the term structure we used the issues presents in tables 5-9.

Table 5

Colombia 2012 Sovereign Bond

Issuer	REPUBLIC OF COLOMBIA
Country	Colombia
Maturity	January 23, 2012
Coupon	Fixed 10% Semi Annual
Day Count	30/360
Fitch Rating	BB+
Yield (3/31/2008)	4,577%

Source: Bloomberg (s. f.).

Table 6

Colombia 2013 Sovereign Bond

Issuer	REPUBLIC OF COLOMBIA
Country	Colombia
Maturity	January 15, 2013
Coupon	Fixed 10.75% Semi Annual
Day Count	30/360
Fitch Rating	BB+
Yield (3/31/2008)	4.963%

Source: Bloomberg (s. f.).

Table 7

Colombia 2014 Sovereign Bond

Issuer	REPUBLIC OF COLOMBIA
Country	Colombia
Maturity	December 22, 2014
Coupon	Fixed 11.75% Semi Annual
Day Count	30/360
Fitch Rating	BB+
Yield (3/31/2008)	5.404%

Source: Bloomberg (s. f.).

Table 8

Colombia 2017 Sovereign Bond

Issuer	REPUBLIC OF COLOMBIA
Country	Colombia
Maturity	January 27, 2017
Coupon	Fixed 7.375% Semi Annual
Day Count	30/360
Fitch Rating	BB+
Yield (3/31/2008)	5.803%

Source: Bloomberg (s. f.).

Table 9

Colombia 2020 Sovereign Bond

Issuer	REPUBLIC OF COLOMBIA
Country	Colombia
Maturity	February 25, 2020
Coupon	Fixed 11.75% Semi Annual
Day Count	30/360
Fitch Rating	BB+
Yield (3/31/2008)	6.091%

Source: Bloomberg (s. f.).

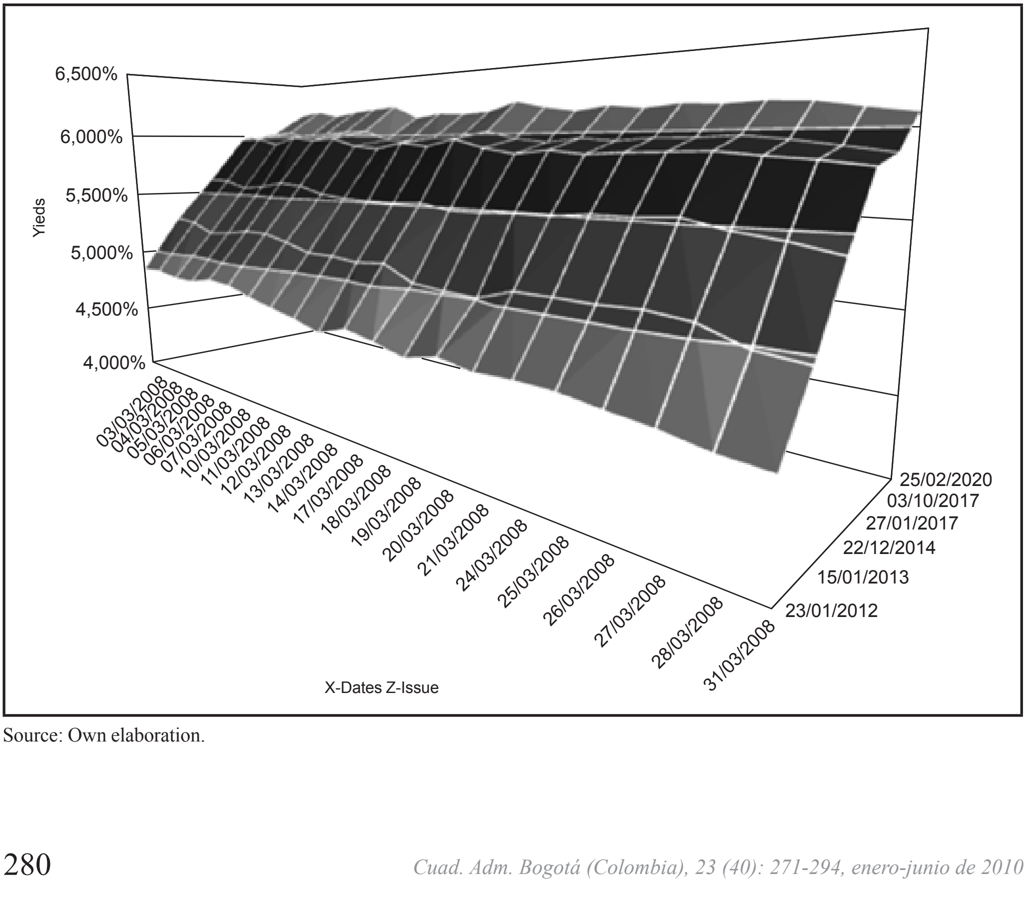
Once the optimal parameters in the Nelson Siegel were found for the date 3/31/2008 by making (t) equal to the time left to maturity of TGI (9.50555 years) we found that the theoretical yield for a Colombian sovereign bond with the same maturity date as TGI using Nelson and Siegel was approximately 5.867%, the difference between the rate found using Nelson and Siegel and that found by using a simpler linear interpolation was just 0.0002%. In average, for the observed period of one year, the difference between the results obtained by simple linear interpolation and Nelson and Siegel was just 0.0187%. The behavior of the intertemporal term structure of the Colombian sovereign bonds can be observed in Figure 3.

2.2 *Step 2-Theoretical Colombian Sovereign Bond Yields as a Proxy Variable for Volatility Estimates*

Once we found the approximate theoretical yield of a non-callable Colombian sovereign bond, we can use the same process for creating a synthetic historical series in order to

Figure 3

Intertemporal Yield Curve for Colombian sovereign bonds-March 2008 including the Theoretical Yield of an Issue dated 03-10-2017 using N&S



Source: Own elaboration.

measure the behavior of the volatility of that theoretical bond in the past. The dataset[5] for obtaining the theoretical yields was formed by the historical closing prices and yield observations of the 2017 and 2020 Colombian sovereign issues from March 30, 2007 until March 30, 2008. Nelson and Siegel was used to obtain a theoretical yield was found for each observation that comprised the dataset. Once the yield was obtained, we found the clean price of the theoretical bond for each date. The summary of the historical price and yield behavior for the two sovereign bonds as well as the theoretical bond are compared in figures 4 and 5.

Since the yield is the determinant of price in a bond, we proceeded to calculate the volatility of the yield of the theoretical bond in the following way, on the assumption that the yields are continuously compounded:

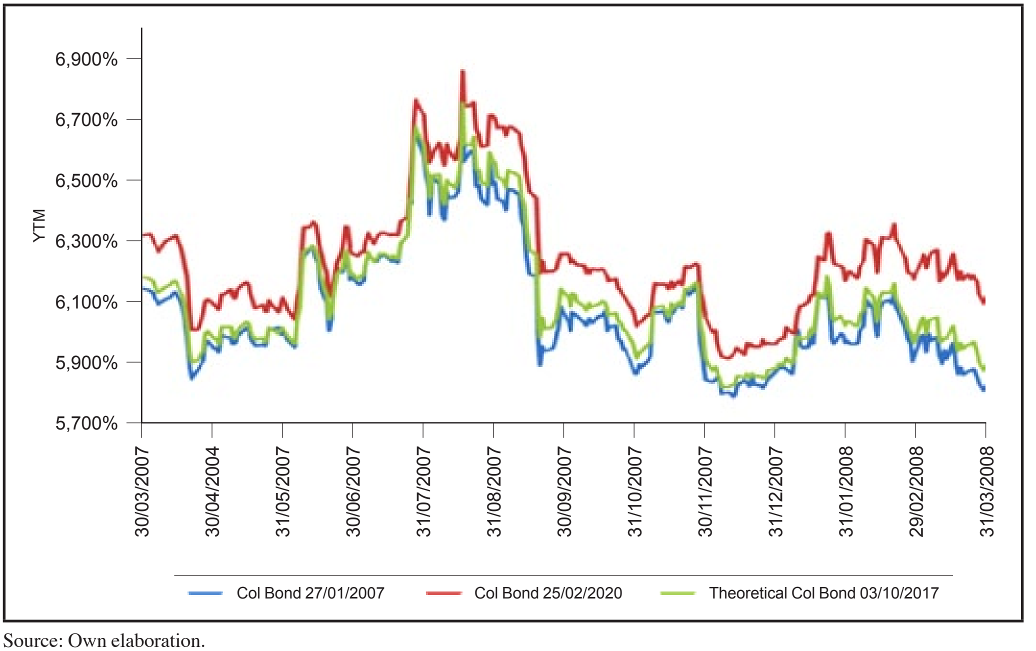
Daily yield variation is found using the following formula:

$$Y\% = \ln\left(\frac{Y_t}{Y_{t-1}}\right)$$

Once we have found the daily yield variations, we can calculate the daily volatility

Figure 4

Historical Real vs. Theoretical Yield (N&S) Comparison 30/03/2007-31/03/2008



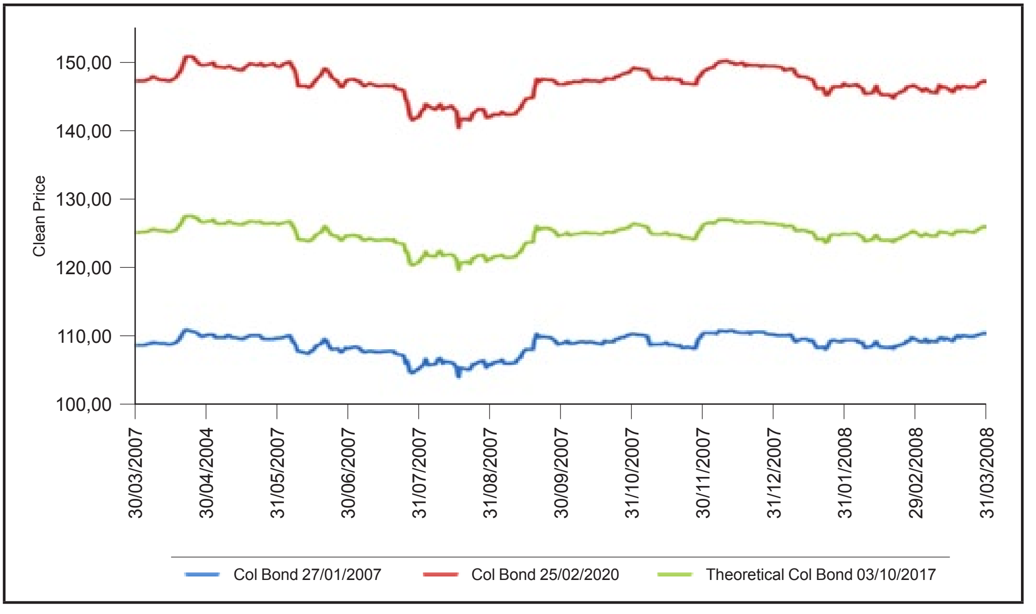
Source: Own elaboration.

measured by standard deviation using the following formula:

[5] Each dataset was comprised of 262 observations. *Source: Bloomberg.*

Figure 5

Historical Real vs. Theoretical Clean Price Comparison 30/03/2007-31/03/2008



Source: Own elaboration.

$$\sigma = \sqrt{\frac{1}{n}\sum_{n=1}^{n}(Y\% - \overline{Y\%})}$$

Where n is the number of observations in the dataset and Y% is the average daily volatility.

For our specific example our daily volatility is equal to 0.702539% since the effective trading days for the bonds were 262 and assuming constant volatility we can turn our daily volatility into annual volatility in the following way:

$$\sigma_{year} = \sigma_{daily} * \sqrt{262}$$

Therefore our annual standard deviation is 11.372%, and we can obtain the semiannual volatility in the following way:

$$\sigma_{semiannual} = \sigma_{year} * \sqrt{1/2}$$

The semiannual volatility for our theoretical sovereign bond would be 8.04093%, also because we know that there are 3 days for the next semiannual coupon in the TGI case using the same formula we find that the expected volatility for the next three days is equal to 1.21683%. Given the fact that sovereign bonds of emerging markets do not trade frequently, reliance on historical prices alone can lead to over-or under-estimation of the volatility of the bond. In order to correct this distortion so we can obtain a better estimate of the theoretical sovereign bond real volatility, we used the EWMA (Exponentially Weighted Moving Average) model for our volatility estimation (Riskmetrics, 1996) the formula is:

$$\sigma = \sqrt{(1-\lambda)\sum_{n=1}^{n}\lambda^{t-1}(Y\% - \overline{Y\%})^2}$$

In order to estimate the optimal decay factor (λ) we minimized the RMSE resultant of an initial decay factor of 0.90, and the optimal decay factor (λ) for the period under observation was 0.9982. Unlike yield, where the difference between a simple linear interpolation and Nelson and Siegel was practically insignificant, in the case of volatility the differences between the two methods are significant. By using EWMA the forecast for annual volatility on 3/31/2008 was 7.1342959%, on a semiannual basis it was 5.0447090% and the expected volatility for the next three days was 0.763416%. Given the fact that most of the research on volatility tends to point out that "historic volatility" is the worst predictor of future volatility (Alexander, 2001), we choose the EWMA as the model for the volatility estimates in the present study. Another reason is the fact that since the EWMA model takes gives more weight to the latest observations and to some extent it helps to correct the problems concerning the liquidity of the Colombian sovereign bond market.

2.3 *Step 3-Constructing a Lattice Using the Theoretical Colombian Sovereign Bond Yield Data and Observed Volatility*

If, for purposes of simplicity, we assume that the yields follow a log-normal distribution (because like prices, yields can never be below zero), then the upward factor required to construct the lattice would be the geometric standard deviation[6] of the synthetic series or exp(σ); and likewise the downward factor will be the inverse mean or (1/ exp(σ)). Of course this approach for determining the factors assumes that there is no significant variation on the median yield over the life of the option (an assumption that is often violated in practice). Also, a more practical approach would be to use a subjective upward and downward factor based on our feelings about the behavior of the market for the period under study (Wong, 1993). It is important to remember that the yield and the volatility used in this example were those estimated by using Nelson and Siegel and the EWMA as proposed by Riskmetrics.

Therefore, by applying the formula for the geometric standard deviation in our previous results, we can find the expected semiannual and three days volatility for theoretical issue, and the results are in Table 10.

[6] The geometric standard deviation is defined as the exponentiated value of the standard deviation of the log transformed values

Table 10

Binomial Price Lattice Data

Yield Volatility Theoretical Bond (Fractional)	0.7634%
Upward factor	1.007663372
Downward factor	0.992394909
Yield Volatility Theoretical Bond (Semi Annual)	5.045%%
Upward factor	1.051741214
Downward factor	0.950804234

Note: The upward and downward factors are calculated using exp(σ) and (1/ exp(σ)) were σ is the yield volatility for both the fractional and semiannual periods.

Source: Own elaboration.

Using the upward and downward factors we can construct the lattice starting from our semiannual theoretical yield of (5.867%/2) = 2.934%. Since the date of the valuation is March 31, 2008 and the next coupon date is April 3, 2008 the upward and downward expected yields for that specific date in the lattice would be 2.934% × 1.007663372 = 2.956% and 2.934% × 0.992394909 = 2.93352%[7] respectively. For the dates of October 3, 2008 onwards we use the semiannual factors using our previous yields in the lattice. Therefore for that specific date the yields are 2.956% × 1.051741214 = 3.1089% and 2.93352% × 1.051741214 = 2.8106% for the upward branches, for the downward branches the results are 2.956% × 0.950804234 = 2.8106% and 2.93352% × 0.950804234 = 2.7892%. The summary of the results are shown in Table 11.

2.4 *Step 4-Finding a Theoretical Discounted Non-Callable Sovereign Bond Price Lattice Using the Future Expected Yield Behavior Lattice*

The first step in finding the discounted non-callable sovereign bond price is to calculate the risk-neutral probabilities for a replicating portfolio at each node. The upward and downward risk neutral probabilities are found using the semiannual and three days observed theoretical rate of 2.934% and 0.049% = (2,934% × 3/180) as follows:

Upward risk-neutral semiannual probability = (1 + 2.934% – 0.950804234)/ (1.051741214 – 0.950804234)= 77.802%

Downward risk-neutral semiannual probability =1 – 77.802% = 22.198%

The same procedure is applied to the three days rate and factors:

Upward risk-neutral semiannual probability = (1 + 0.049% – 0.992394909)/ (1.007663372 – 0.992394909)= 53.011%

Downward risk-neutral semiannual probability =1 – 53.011% = 46.989%

The theoretical price of the bonds is found discounting the principal and the coupons independently in a backward manner. As we can observe form the yield lattice on April 3, 2017 we have a total of 210 possible branches (or expected yields). For the date of October 3, 2008 or the date of expiration of the bond we can expect to receive a notional principal of 100 for the 21 possible branches

7 The results in the lattice are rounded up to three decimal places, so 2.93352% would be presented as 2.934% in the lattice.

Table 11

Theoretical Yield Lattice

	31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	**03/10/2012**	03/04/2013	**03/10/2013**	03/04/2014	**03/10/2014**	03/04/2015	**03/10/2015**	03/04/2016	03/10/2016	03/04/2017	03/10/2017
Fraccionate Semianual Periods		0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Semianual Rates	2,934%	2,956%	3,109%	3,270%	3,439%	3,617%	3,804%	4,001%	4,208%	4,426%	4,655%	4,895%	5,149%	5,415%	5,695%	5,990%	6,300%	6,626%	6,969%	7,329%	
		2,934%	2,811%	2,956%	3,109%	3,270%	3,439%	3,617%	3,804%	4,001%	4,208%	4,426%	4,655%	4,895%	5,149%	5,415%	5,695%	5,990%	6,300%	6,626%	
			2,789%	2,672%	2,811%	2,956%	3,109%	3,270%	3,439%	3,617%	3,804%	4,001%	4,208%	4,426%	4,655%	4,895%	5,149%	5,415%	5,695%	5,990%	
				2,652%	2,541%	2,672%	2,811%	2,956%	3,109%	3,270%	3,439%	3,617%	3,804%	4,001%	4,208%	4,426%	4,655%	4,895%	5,149%	5,415%	
					2,522%	2,416%	2,541%	2,672%	2,811%	2,956%	3,109%	3,270%	3,439%	3,617%	3,804%	4,001%	4,208%	4,426%	4,655%	4,895%	
						2,397%	2,297%	2,416%	2,541%	2,672%	2,811%	2,956%	3,109%	3,270%	3,439%	3,617%	3,804%	4,001%	4,208%	4,426%	
							2,280%	2,184%	2,297%	2,416%	2,541%	2,672%	2,811%	2,956%	3,109%	3,270%	3,439%	3,617%	3,804%	4,001%	
								2,167%	2,077%	2,184%	2,297%	2,416%	2,541%	2,672%	2,811%	2,956%	3,109%	3,270%	3,439%	3,617%	
									2,061%	1,974%	2,077%	2,184%	2,297%	2,416%	2,541%	2,672%	2,811%	2,956%	3,109%	3,270%	
										1,959%	1,877%	1,974%	2,077%	2,184%	2,297%	2,416%	2,541%	2,672%	2,811%	2,956%	
											1,863%	1,785%	1,877%	1,974%	2,077%	2,184%	2,297%	2,416%	2,541%	2,672%	
												1,771%	1,697%	1,785%	1,877%	1,974%	2,077%	2,184%	2,297%	2,416%	
													1,684%	1,614%	1,697%	1,785%	1,877%	1,974%	2,077%	2,184%	
														1,601%	1,534%	1,614%	1,697%	1,785%	1,877%	1,974%	
															1,523%	1,459%	1,534%	1,614%	1,697%	1,785%	
																1,448%	1,387%	1,459%	1,534%	1,614%	
																	1,376%	1,319%	1,387%	1,459%	
																		1,309%	1,254%	1,319%	
																			1,244%	1,192%	
																				1,183%	

Source: Own elaboration.

on that specific date, in the same way as the principal, we can expect to receive a coupon of 4.75. As observed from the yield lattice in April 3, 2017 the highest yields expected in the upward branches are 7.329% and 6.626% respectively. Therefore, the expected principal price for those yields in April 3, 2017 are 100/(1 + 7.329%) = 93.17116911 and 100/(1 + 6.626%) = 93.78581492. In this way we can find the expected price for the upward branch on October 3, 2016 by discounting the expected prices for April 3, 2017 and applying the risk-neutral semiannual probability for each price in the following way:

Expected price on October 3, 2016 = (77.802% × (93.17116911/(1 + 6.969%[8])) + (22.198% × (93.78581492/(1+6.300%)) = 87.35128424

For the coupons the procedure is the same as that used for the principal with the difference that we accrue the coupons of each period. From the yield lattice, we can observe that in April 3, 2017 the highest yields expected in the upward branches are 7,329% and 6,626% respectively. Therefore, the expected accrued coupon prices for those yields on April 3, 2017 are (4.75/(1 + 7.329%)) + 4.75 = 9.175630533 and (4.75/(1+6.626%)) + 4.75 = 9.204826209. In this way we can find the expected accrued coupon prices for the upward branch on October 3, 2016 by discounting the expected accrued coupon prices for April 3, 2017 and applying the risk-neutral semiannual probability for each price in the following way:

Expected accrued coupon price on October 3, 2016 = (77.802% × (9.175630533/(1 + 6.969%) + 4.75)) + (22.198% × (9.204826209/(1 + 6.300% + 4.75))= 13.3459363

In this way, we continue to value the principal backwards to April 3, 2008 for valuing the principal and the coupons on the date of March 31, 2008, we use the risk three-day neutral probability and the fractional discount factor for the period (3/180 = 0.01666667) as follows:

Expected price on March 31, 2008 = (53.011% × (49.32627606/(1 + 2.934%)^0.01666667)) + (46.989% × (52.9608102/(1 + 2.934%)^0.01666667))= 51.02149867

Expected accrued coupon price on March 31, 2008 = ((53.011% × ((70.26506064/(1 + 2.934%)^0.01666667)) + 4.75))+((46.989% × ((72.45660412/(1+ 2.934%^0.01666667)) + 4.75))= 76.02689315

The expected non-callable price for the theoretical bond would be the sum of the expected price for the principal and coupons on March 31, 2008 that means that the expected non-callable price would be 51.02149867 + 76.02689315 = 127.0483918. In the same way, a theoretical price can be found for each node of the non-callable bond price lattice. In tables 12, 13, and 14 we can observe a summary of the results for the principal, coupons and expected bond prices.

8 The yields used to discount this node are those in the upward branches of the yield lattice for October 3, 2016.

Table 12

Discounted Expected Principal Price Lattice

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
51,0214987	49,3262761	50,0441165	50,82494165	51,7166677	52,7310384	53,8814572	55,1832736	56,654126	58,3143553	60,18750299	62,3009171	64,68648906	67,3815552	70,43000483	73,8836476	77,80390912	82,2639443	87,3512842	93,1711691	100
	52,9608102	53,3671986	54,15916349	55,0184415	55,9942863	57,0989657	58,3464529	59,7527158	61,3360639	63,11756303	65,1215368	67,37617196	69,914254	72,77406409	76,0004781	79,64631773	83,7740212	88,4577176	93,7858149	100
		56,8671472	57,33654734	58,1987806	59,1322105	60,1871045	61,3761501	62,7137694	64,2164096	65,90288821	67,7948074	69,91705146	72,2983865	74,97218728	77,9773208	81,3592247	85,1712297	89,476189	94,3484937	100
			60,67501303	61,2048504	62,1325358	63,1348091	64,2626342	65,5290123	66,9486938	68,53846495	70,3174907	72,30772275	74,5343893	77,02658406	79,8179773	82,9476778	86,4612822	90,4121573	94,8630141	100
				64,3599832	64,9468299	65,9343846	66,9993954	68,1932176	69,529063	71,02189109	72,6886921	74,54882233	76,6244041	78,94080335	81,527203	84,41729277	87,6501035	91,271019	95,3330089	100
					67,9017748	68,5415125	69,5827814	70,7038368	71,9561382	73,35300814	74,9095025	76,64268658	78,5719617	80,71945313	83,110473	85,77407346	88,7437115	92,0580495	95,7619239	100
						71,2841942	71,9721773	73,0606207	74,2306423	75,53354008	76,9826538	78,59303057	80,3816916	82,36794805	84,5737754	87,02425772	89,7481188	92,7783574	96,1530106	100
							74,4949617	75,2261739	76,3550408	77,56675071	78,9121978	80,40465365	82,0590602	83,89228683	85,9234332	88,17418795	90,6692532	93,4368503	96,5093236	100
								77,5254449	78,2946441	79,4571277	80,7032137	82,08317601	83,610141	85,29885975	87,1659537	89,23020445	91,5128942	94,0382098	96,8337212	100
									80,3703295	81,17217683	82,3615491	83,63480749	85,0414151	86,59428527	88,3079031	90,19855947	92,2846254	94,5868755	97,1288686	100
										83,02725205	83,8564246	85,06614871	86,3596027	87,78527393	89,3558045	91,08534949	92,9897986	95,0870357	97,397243	100
											85,4964173	86,34770161	87,5715228	88,87851717	90,3160569	91,8964636	93,6335088	95,5426233	97,6411408	100
												87,7802172	88,6485844	89,88059852	91,1948724	92,63754544	94,2205785	95,9573165	97,8626851	100
													89,8828677	90,76352908	91,9982285	93,31396687	94,7555491	96,3345434	98,0638345	100
														91,81007871	92,6985284	93,9308111	95,2426786	96,6774885	98,2463924	100
															93,5687425	94,46086818	95,6859433	96,9891022	98,4120159	100
																95,16637475	96,0592336	97,2721108	98,5622256	100
																	96,608233	97,5052902	98,6984147	100
																		97,8788686	98,8218579	100
																			98,830713	100
																				100

Source: Own elaboration.

Table 13

Discounted Coupon Expected Price Lattice

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
76,0268932	70,2650606	67,0633575	63,8384958	60,6336376	57,4441305	54,2646413	51,0890528	47,9103415	44,7204309	41,51001521	38,2683471	34,98297959	31,6394529	28,22091113	24,7076315	21,07644417	17,3000124	13,3459363	9,17563053	4,75
	72,4566041	68,797722	65,50510002	62,1872206	58,8834613	55,5887364	52,2972245	49,0022569	45,6961871	42,37023317	39,0142905	35,61670496	32,1640003	28,64054616	25,0281545	21,30558512	17,447938	13,4259026	9,20482621	4,75
		70,7543363	67,03514575	63,6502423	60,2378497	56,8335918	53,431954	50,0266334	46,610421	43,17506247	39,7110923	36,20763513	32,6521682	29,0302347	25,3250967	21,51731471	17,5842335	13,499353	9,23155345	4,75
			68,76691212	64,9859481	61,5078506	57,9999386	54,4941404	50,9845228	47,4643159	43,92578931	40,3601057	36,75714731	33,1053084	29,39124778	25,5995913	21,71257249	17,7095986	13,5667216	9,25599317	4,75
				66,5049038	62,6610057	59,0892796	55,4853595	51,87756	48,2595458	44,62410121	40,9630005	37,26685622	33,5249385	29,72496129	25,8528262	21,89231653	17,8247294	13,628431	9,27831792	4,75
					63,9806896	60,0730339	56,4077353	52,7078468	48,9981729	45,27199817	41,5216873	37,73855202	33,9126939	30,03281928	26,086019	22,05750754	17,9303101	13,6848888	9,29869139	4,75
						61,2080526	57,2361727	53,4778466	49,6825547	45,87171244	42,0382505	38,17414563	34,2702844	30,31630249	26,3003956	22,20909578	18,0270063	13,7364853	9,31726801	4,75
							58,2017997	54,1655933	50,3152603	46,4256377	42,5148894	38,57562146	34,599458	30,57690191	26,4971718	22,34801062	18,1154599	13,7835919	9,33419287	4,75
								54,9774097	50,8771362	46,93626774	42,9538677	38,94499733	34,9019707	30,8160971	26,6775398	22,47515233	18,1962854	13,8265603	9,34960176	4,75
									51,5507169	47,38698522	43,3574715	39,28429123	35,1795615	31,03533875	26,8426569	22,59138602	18,2700674	13,8657215	9,36362126	4,75
										47,9376345	43,7113833	39,59549435	35,4339322	31,236035	26,9936369	22,69753717	18,3373591	13,9013858	9,37636904	4,75
											44,1539167	39,86638978	35,6667326	31,41954115	27,131544	22,79438874	18,3986808	13,9338432	9,38795419	4,75
												40,2149624	35,8676791	31,5871522	27,2573889	22,88267949	18,4545205	13,9633636	9,39847754	4,75
													36,1356556	31,73037928	27,3721261	22,96310321	18,5053337	13,9901975	9,40803214	4,75
														31,93024171	27,4689466	23,03630887	18,5515444	14,014577	9,41670364	4,75
															27,6122331	23,09707654	18,593546	14,036716	9,42457075	4,75
																23,19432661	18,6276538	14,056812	9,43170572	4,75
																	18,6882304	14,0727779	9,4381747	4,75
																		14,1038258	9,44403825	4,75
																			9,44445887	4,75
																				4,75

Source: Own elaboration.

Table 14

Theoretical Expected Non-callable Bond Prices-(The Sum of Table 12 and 13 for each node)

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
127,048392	119,591337	117,107474	114,6634374	112,350305	110,175169	108,146098	106,272326	104,564468	103,034786	101,6975182	100,569264	99,66946865	99,0210081	98,65091597	98,5912791	98,8803533	99,5639567	100,697221	102,3468	104,75
	125,417414	122,164921	119,6642635	117,205662	114,877748	112,687702	110,643677	108,754973	107,032251	105,4877962	104,135827	102,9928769	102,078254	101,4146102	101,028633	100,9519028	101,221959	101,88362	102,990641	104,75
		127,621483	124,3716931	121,849023	119,37006	117,020696	114,808104	112,740403	110,826831	109,0779507	107,5059	106,1246866	104,950555	104,002422	103,302418	102,8765394	102,755463	102,975542	103,580047	104,75
			129,4419252	126,190798	123,640386	121,134748	118,756775	116,513535	114,41301	112,4642543	110,677596	109,0648701	107,639698	106,4178318	105,417569	104,6602503	104,170881	103,978879	104,119007	104,75
				130,864887	127,607836	125,023664	122,484755	120,070778	117,788609	115,6459923	113,651693	111,8156785	110,149343	108,6657646	107,380029	106,3096093	105,474833	104,89945	104,611327	104,75
					131,882464	128,614546	125,990517	123,411684	120,954311	118,6250063	116,43119	114,3812386	112,484656	110,7522724	109,196492	107,831581	106,674022	105,742938	105,060615	104,75
						132,492247	129,20835	126,538467	123,913197	121,4052525	119,020904	116,7671762	114,651976	112,6842505	110,874171	109,2333535	107,775125	106,514843	105,470279	104,75
							132,696761	129,391767	126,670301	123,9923884	121,427087	118,9802751	116,658518	114,4691887	112,420605	110,5221986	108,784713	107,220442	105,843517	104,75
								132,502855	129,17178	126,3933954	123,657081	121,0281733	118,512112	116,1149569	113,843494	111,7053568	109,70918	107,86477	106,183323	104,75
									131,921046	128,5591621	125,719021	122,9190987	120,220977	117,629624	115,15056	112,7899455	110,554693	108,452597	106,49249	104,75
										130,9648865	127,567808	124,6616431	121,793535	119,0213089	116,349441	113,7828867	111,327158	108,988422	106,773612	104,75
											129,650334	126,2140914	123,238255	120,2980583	117,447601	114,6908523	112,03219	109,476466	107,029095	104,75
												127,9951796	124,516264	121,4677507	118,452261	115,5202249	112,675099	109,92068	107,261163	104,75
													126,018523	122,4939084	119,370355	116,2770701	113,260883	110,324741	107,471867	104,75
														123,7403204	120,167475	116,96712	113,794223	110,692065	107,663096	104,75
															121,180976	117,5579447	114,279489	111,025818	107,836587	104,75
																118,3607014	114,686887	111,328923	107,993931	104,75
																	115,296463	111,578068	108,136589	104,75
																		111,982694	108,265896	104,75
																			108,275172	104,75
																				104,75

Source: Own elaboration.

2.5 *Step 5-Finding a Theoretical Call Price for Each Option Embedded in the Callable Bond Using the Theoretical Non-Callable Sovereign Bond Price Lattice*

Once we have the expected non-callable price for each node until maturity we can proceed to calculate the theoretical value for each option embedded on the bond according to the following call schedule (Table 15).

Table 15

Call Schedule

Date (mm/dd/yyyy)	Exercise Price
10/03/2012	104.750
10/03/2013	103.167
10/03/2014	101.583
10/03/2015	100.000

Source: Bloomberg (s. f.).

Since the call is priced backwards we begin with the first option that has an exercise price of 104.75 on October 3, 2012. As we can appreciate from the non-callable bond price lattice from the possible 11 expected prices on October 3, 2012, just 8 of them will be in the money, or have an exercise price that is greater than the expected price. Therefore, the possible notional call prices on that date would be as follow:

03/10/2012
9

0
0,7377962
4,327950686
7,714254258
10,8959923
13,87500631
16,65525252
19,24238841
21,64339544
23,80916205
26,21488654

If the exercise price is 104.75 and the expected price on the upward node is 101.6975182, then the call price would be cero because C=MAX(0, 101.6975182-104.75). In the case of the second node, the call price would be 0.7377962 because C=MAX(0, 105.4877962-104.75) and so forth until the call price for each node for an expected non-callable price is found. Then the call option is priced backwards using the semiannual risk-neutral probability in the following way:

Expected Call Price second Node on April 3, 2012 = ((77.802% × 0) + (22.198% × 0.7377962))/(1+4.426%[9])=0.156835111

Then we continue to price the call backwards to April 3, 2008. For valuing the call option on March 31, 2008, we use the risk three day neutral probability and the fractionate discount factor for the period (3/180=0.01666667) as follows:

Expected Call Price on March 31, 2008 = ((53.011% × 5.35010251)+ (46.989% × 9.06234989))/(1+2.934%[10])^0.01666667))= 4.521392559

It is important to note that in the nodes where the option is exercised, for the next option

[9] This is the yield found in the first node on April 3, 2012

[10] This is the yield found in the first node on March 31, 2008 in the yield lattice.

only the nodes that were not exercised in the first option will be taken into account when valuing the second option scheduled on October 3, 2013. Therefore, the expected prices used to price the second option would be (note that the paths after the exercise of the first option cease to exist because the bond has been recalled by the issuer through the exercise of the first call option) (Table 16).

Table 16

Call Price Paths

03/10/2012	03/04/2013	03/10/2013
9	10	11
101.697518	100.569264	99.6694686
Exercise	104.135827	102.992877
Exercise	0	106.124687
Exercise	0	0
Exercise	0	0
Exercise	0	0
Exercise	0	0
Exercise	0	0
Exercise	0	0
Exercise	0	0
	0	0
	0	0
		0

Source: Own elaboration.

If the second option exercise price on October 3, 2013 is 103.167 and we just have two expected prices for that date, then the notional call prices for the second option would be:

03/10/2013
11
0
0
2,957686587
0
0
0
0
0
0
0
0
0
0

If the stated price for that date is greater than the exercise price of the option of 103.167, the option will be exercised: otherwise the option will be allowed to expire and its value would be zero. With these notional call values, we use the same procedure of the first option to find the value of the second option on March 31, 2008. The third and fourth option call values are found in the same way as the second option (taking into account only the stated prices that have not been exercised in the previous option until the last option expires). The results for the four options are shown in tables 17 to 20:

Therefore, by adding the four option call prices we found that the embedded options of the bond have a total value of *4.521392559 + 0.496152948 + 0.487039616 + 0.122441017 = 5.62702614*.

2.6 *Step 6-Finding the Theoretical Option Adjusted Spread for a Theoretical Colombian Sovereign Non-Callable Bond*

Since we know that the theoretical dirty price of a Colombian sovereign bond wi-

Table 17

First call lattice 10/03/2012 Strike 104.75

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	**03/10/2012**	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
4,52139256	3,38475011	2,94295361	2,4967768	2,04998787	1,60855306	1,18157211	0,78259054	0,43144166	0,15683511	0	0	0	0	0	0	0	0	0	0	0
	5,80835809	5,38393736	4,918938827	4,43051125	3,91476789	3,367179	2,78245222	2,15438709	1,47569776	0,7377962	0	0	0	0	0	0	0	0	0	0
		8,06350273	7,695396705	7,28586426	6,85866124	6,41066984	5,93824816	5,43713435	4,90232877	4,327950686	0	0	0	0	0	0	0	0	0	0
			10,36686883	10,057182	9,70566442	9,34216508	8,96431304	8,56932182	8,15391908	7,714254258	0	0	0	0	0	0	0	0	0	0
				12,6908187	12,4403913	12,1481143	11,8493511	11,5424556	11,2254496	10,8959923	0	0	0	0	0	0	0	0	0	0
					15,0101221	14,8187036	14,5857581	14,3514775	14,1149698	13,87500631	0	0	0	0	0	0	0	0	0	0
						17,3021792	17,1685597	16,9942824	16,8231245	16,65525252	0	0	0	0	0	0	0	0	0	0
							19,5472748	19,4685475	19,3527024	19,24238841	0	0	0	0	0	0	0	0	0	0
								21,7317772	21,6957923	21,64339544	0	0	0	0	0	0	0	0	0	0
									23,8753762	23,80916205	0	0	0	0	0	0	0	0	0	0
										26,21488654	0	0	0	0	0	0	0	0	0	0
											0	0	0	0	0	0	0	0	0	0
												0	0	0	0	0	0	0	0	0
													0	0	0	0	0	0	0	0
														0	0	0	0	0	0	0
															0	0	0	0	0	0
																0	0	0	0	0
																	0	0	0	0
																		0	0	0
																			0	0
																				0

Source: Own elaboration.

Table 18

Second call lattice 10/03/2013 Strike 103.167

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0,49615295	0,5739548	0,61938962	0,654470216	0,67330488	0,66896294	0,63401516	0,56177985	0,44879153	0,29933015	0,133356634	0	0	0	0	0	0	0	0	0	0
	0,40888806	0,49114073	0,583184152	0,68486117	0,79283377	0,90045217	0,99584458	1,05904715	1,05771412	0,940729901	0,62872255	0	0	0	0	0	0	0	0	0
		0,17463556	0,230722245	0,30447532	0,40234554	0,53242709	0,70561174	0,93658773	1,24520839	1,658372375	2,21261448	2,957686587	0	0	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
					0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	0	0	0	0	0	0	0	0	0	0	0	0	0
								0	0	0	0	0	0	0	0	0	0	0	0	0
									0	0	0	0	0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0	0	0	0	0
											0	0	0	0	0	0	0	0	0	0
												0	0	0	0	0	0	0	0	0
													0	0	0	0	0	0	0	0
														0	0	0	0	0	0	0
															0	0	0	0	0	0
																0	0	0	0	0
																	0	0	0	0
																		0	0	0
																			0	0
																				0

Source: Own elaboration.

Table 19

Third call lattice 10/03/2014 Strike 101.583

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0,48703962	0,62842685	0,69404137	0,754786074	0,80560626	0,84008885	0,85054129	0,82840963	0,765303	0,65505316	0,497486966	0,30497202	0,113379754	0	0	0	0	0	0	0	0
	0,32753057	0,39845397	0,481136741	0,57666598	0,68474794	0,80345399	0,92811072	1,0495926	1,15171863	1,207308247	1,17223423	0,976485412	0,51076542	0	0	0	0	0	0	0
		0,12223452	0,161718609	0,21370126	0,28279261	0,37477766	0,49745794	0,6613783	0,88082566	1,175205061	1,57094165	2,104117799	2,82414165	2,419421982	0	0	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	4,834831839	0	0	0	0	0	0
				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
					0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	0	0	0	0	0	0	0	0	0	0	0	0	0
								0	0	0	0	0	0	0	0	0	0	0	0	0
									0	0	0	0	0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0	0	0	0	0
											0	0	0	0	0	0	0	0	0	0
												0	0	0	0	0	0	0	0	0
													0	0	0	0	0	0	0	0
														0	0	0	0	0	0	0
															0	0	0	0	0	0
																0	0	0	0	0
																	0	0	0	0
																		0	0	0
																			0	0
																				0

Source: Own elaboration.

Table 20

Fourth call lattice 10/03/2015 Strike 100

31/03/2008	03/04/2008	03/10/2008	03/04/2009	03/10/2009	03/04/2010	03/10/2010	03/04/2011	03/10/2011	03/04/2012	03/10/2012	03/04/2013	03/10/2013	03/04/2014	03/10/2014	03/04/2015	03/10/2015	03/04/2016	03/10/2016	03/04/2017	03/10/2017
	0,01666667	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
0,12244102	0,19749684	0,23959004	0,289084109	0,34656346	0,4122588	0,48576326	0,56560082	0,64857989	0,72883551	0,79642241	0,83526191	0,820161427	0,71250515	0,454042329	0,21130348	0	0	0	0	0
	0,03776518	0,04996405	0,066117833	0,08762424	0,11630729	0,1546324	0,20593991	0,27476678	0,36729128	0,491949602	0,66029353	0,888187704	1,19748683	1,618393717	1,3048198	0,951902848	0	0	0	0
		0	0	0	0	0	0	0	0	0	0	0	0	0	3,11224569	2,876539418	0	0	0	0
			0	0	0	0	0	0	0	0	0	0	0	0	0	4,660250288	0	0	0	0
				0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
					0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
						0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
							0	0	0	0	0	0	0	0	0	0	0	0	0	0
								0	0	0	0	0	0	0	0	0	0	0	0	0
									0	0	0	0	0	0	0	0	0	0	0	0
										0	0	0	0	0	0	0	0	0	0	0
											0	0	0	0	0	0	0	0	0	0
												0	0	0	0	0	0	0	0	0
													0	0	0	0	0	0	0	0
														0	0	0	0	0	0	0
															0	0	0	0	0	0
																0	0	0	0	0
																	0	0	0	0
																		0	0	0
																			0	0
																				0

5,62702614

Source: Own elaboration.

th a coupon of 9.5% on March 31, 2008 is 127.0483918. Also, we know that the value of the call option in the hands of the issuer is *5.62702614*. Therefore, the expected dirty price on March 31, 2008 of a theoretical callable Colombian sovereign bond with the same maturity, coupon and call schedule as TGI would be 127.0483918 – 5.62702614 = 121.4213657. If the bond pays a 4.75% semiannual coupon on April 3, 2008 on a 30/360 basis then the accrued interest up to that date would be ((4.5%/180)×177) = 4.67083333. Therefore, the clean price of our theoretical callable bond would be 122.768876-4.67083333=116.7505323 and the expected yield of a theoretical Colombian sovereign callable bond would be as Table 21.

Table 21

Theoretical Colombian Sovereign Callable Bond

Liquidation date	31/03/2008
Settlement	03/10/2007
Coupon	9.50%
Principal	100
Clean price	116.750532
Yield	7.052%

Note: The yield is calculated using our theoretical clean price for the liquidation date

Source: Own elaboration.

If we know that the spread of a Theoretical non-callable Colombian Sovereign Bond on March 31, 2008 is 5.867% then 7.052% – 5.867%= 1.185% or approximately 118.85 basic points are attributable to the value of the call options that the investor in theory "sells" to the issuer which is the value of the OAS in this specific example. Similarly, if we know that on March 31, 2008 the market yield of TGI is 8.872%, and we already know the theoretical OAS for a Theoretical Colombian sovereign bond, then we can assume that the difference in spread can be attributable to the company-specific risk of a natural gas company operating in Colombia. In this case this risk can be valued as an additional spread of 8.872%–7.052%=1.820% or approximately 182 basics points. For investment strategy purposes, if we can assume that the company specific risk is constant and that changes in yield are attributable to the country risk and the OAS of the bond on a following date, then we can verify if the callable bond is overpriced or underpriced on that date depending on the expected theoretical OAS or country risk variation.

Conclusions

This paper presents a complete detailed methodological approach for valuing callable bonds in Emerging Markets. Through the development of a practical example using the binomial pricing model, it was possible to determine what the theoretical value of the Option Adjusted Spread of TGI would be. Moreover, by using meaningful proxy variables taken from real-life data, it is possible to find better estimates of the spread attributable to specific risk of companies operating in emerging markets. Although, it is important to remember that in periods of high volatility or market unrest in which the value of the option increases or decreases in an abrupt manner, it would be possible to obtain a negative country risk premium. However, the question remains, whether in a time of market turbulence this premium changes in a manner which is positively correlated with the Colombian sovereign bond discount rate. Also, and of special importance, there is the determination of a theoretical sovereign price for a bond that has the same country of origin as the company whose callable bond issue we wish to value. Another important question for future research is to compare the consistency of the results obtained using Nelson and Siegel vs. other term structure models such as Vasicek or its extended version developed by Hull and White. Finally, by applying a commonly-used methodology such as the binomial pricing formula, we expect to prepare the ground for further research on how to develop methodological approaches on how find meaningful proxy variables for complex valuation models using real market data.

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