A Review of DAN2 (Dynamic Architecture for Artificial Neural Networks) Model in Time Series Forecasting[1]

Revisión del modelo DAN2 (Arquitectura dinámica para redes neuronales artificiales) en predicción de series de tiempo[2]

Uma revisão da DAN2 (arquitetura dinâmica para redes neurais artificiais) modo de previsão de séries temporais[3]

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Abstract

Recently, Ghiassi, Saidane and Zimbra [*Int J Forecasting*, vol. 21, 2005, pp. 341-362] presented a dynamic-architecture neural network for time series prediction which performs significantly better than traditional artificial neural networks and the ARIMA methodology. The main objective of this article is to prove that the original DAN2 model can be rewritten as an additive model. We show that our formulation has several advantages: First, it reduces the total number of parameters to estimate; second, it allows estimating all the linear parameters by using ordinary least squares or ridge regression; and, finally, it improves the search for the global minimum of the error function used to estimate the model parameters. To assess the effectiveness of our approach, we estimate two models for one of the time series used as a benchmark when the original DAN2 model was proposed. The results indicate that our approach is able to find models with similar or better accuracy than the original DAN2.

Resumen

Recientemente, Ghiassi, Saidane y Zimbra [*Int J Forecasting*, vol. 21, 2005, pp. 341-362] presentaron una red neuronal artificial de arquitectura dinámica (DAN2) para la predicción de series de tiempo, la cual se desempeña significativamente mejor que las redes neuronales tradicionales y que la metodología ARIMA. El objetivo principal de este artículo es demostrar que el modelo original DAN2 puede reescribirse como un modelo aditivo. Se muestra que la formulación propuesta tiene varias ventajas: se reduce el número total de parámetros por estimar, permite calcular todos los parámetros lineales usando mínimos cuadrados ordinarios y se mejora la búsqueda del óptimo global de la función de error usada para estimar los parámetros del modelo. A fin de confirmar la efectividad de nuestra aproximación, se estimaron dos modelos para una de las series de tiempo usadas como *benchmark* cuando el modelo DAN2 original fue propuesto. Los resultados indican que nuestra aproximación es capaz de encontrar modelos con una precisión similar o mejor respecto al modelo DAN2 original.

Resumo

Recentemente, Ghiassi, Saidane e Zimbra [*Int J Forecasting*, vol. 21, 2005, pp. 341-362] apresentaram uma rede neural artificial de arquitetura dinâmica (DAN2) para a previsão de séries temporais, que se desempenha significativamente melhor que as redes neurais tradicionais e que a metodologia ARIMA. O objetivo principal deste artigo é demonstrar que o modelo original DAN2 pode ser reescrito como um modelo aditivo. Mostra-se que a formulação proposta tem várias vantagens: se reduz o número total de parâmetros por estimar, permite estimar todos os parâmetros lineares usando mínimos quadrados ordinários e melhora-se a busca do ótimo global da função de erro usada para estimar os parâmetros do modelo. A fim de confirmar a efetividade de nossa aproximação, estimaram-se dois modelos para uma das séries temporais usadas como benchmark quando o modelo DAN2 original foi proposto. Os resultados indicam que nossa aproximação é capaz de encontrar modelos com uma precisão similar ou melhor que o modelo DAN2 original.

Keywords

Neural networks (computer science), artificial intelligence, logic programming, time-series analysis.

Palabras clave

Redes neurales (computadores), inteligencia artificial, programación lógica, análisis de series de tiempo.

Palavras chave

Redes neurais (computadores), inteligência artificial, programação lógica, análise de séries temporais.

Introduction

Nonlinear forecasts can be more accurate than linear models when time series exhibit nonlinearities. Research topics in this area vary from development of new forecast models to comparison of models' performance for different time series, with the objective of determining model accuracy under fixed conditions.

Recently, Ghiassi and Saidane (2005) have developed a new type of artificial neural network named DAN2. Also, this model is used for forecasting six nonlinear time series that are commonly used as benchmarks, and compare the accuracy of DAN2 with other models (Ghiassi, Saidane and Zimbra, 2005). Collected evidences suggest that DAN2 performs better than the ARIMA approach and traditional artificial neural networks (Ghiassi, Saidane and Zimbra, 2005). In a later study, Ghiassi, Zimbra and Saidane (2006) use DAN2 for medium term electrical load forecasting and compare forecasts obtained using DAN2 with forecasts calculated using an ARIMA and a multilayer perceptron. Ghiassi et al. conclude that DAN2 outperforms the competitive models. Gomes, Maia, Ludermir, Carvalho, and Araujo (2006) propose a new forecasting model based on combining the forecasts obtained using DAN2 and the ARIMA approach; the evidences presented indicate that the new model outperforms the individual forecasts obtained with DAN2 and ARIMA. More recently, Ghiassi, Zimbra, and Saidane (2008) compare the performance of DAN2, traditional artificial neural networks and the ARIMA approach for forecasting the short-, medium-, and long-term urban water demand; this study demonstrates the effectiveness of this new class of neural network.

Wang, Niu, and LI (2010) forecast regional electricity load in Taiwan and conclude that DAN2 performs better than regression models, artificial neural networks and support vector machines. Also, DAN2 has been used to forecast electricity prices; Velasquez and Franco (2010) compare the performance of DAN2 with the ARIMA approach when the prices of electricity contracts in the Colombian energy market are forecasted; they conclude that DAN2 is more accurate than the ARIMA model. Guresen, Kayakutlu, and Daim (2011) forecast the NASDAQ index using several nonlinear models including DAN2. Other experiences with DAN2 indicate that this model may be used to solve nonlinear regression (Ghiassi and Nangoy, 2009) and classification (Ghiassi and Burnley, 2010) problems, too.

We propose a modification of the DAN2 model, aimed at improving the network's estimation procedure. In this paper, we revise the mathematical formulation of DAN2, and we prove that the model can be rewritten as an additive model, reducing the number of parameters to be estimated. In Section 1, we revise the architecture proposed for DAN2 and the training algorithm developed by Ghiassi and Saidane (2005). Then, in Section 2, we review the mathematical formulation of DAN2 and propose a training algorithm that simultaneously estimates all linear parameters of the modified DAN2 model. In Section 3, we compare the performance of our approach to the original DAN2 model fitting both models to one of the original time series benchmarks. Section 4 presents our conclusions.

1. The Original DAN2 Model

1.1. DAN2 Original Architecture

The DAN2 architecture, developed by Ghiassi and Saidane (2005) and Ghiassi, Saidane and Zimbra (2005), is described in this section. The main features of the DAN2 model are depicted in Figure 1. As shown in Figure 1, inputs are presented to the network through the input node *I* all at once, and not in the sequential process that is the common practice in neural network literature.

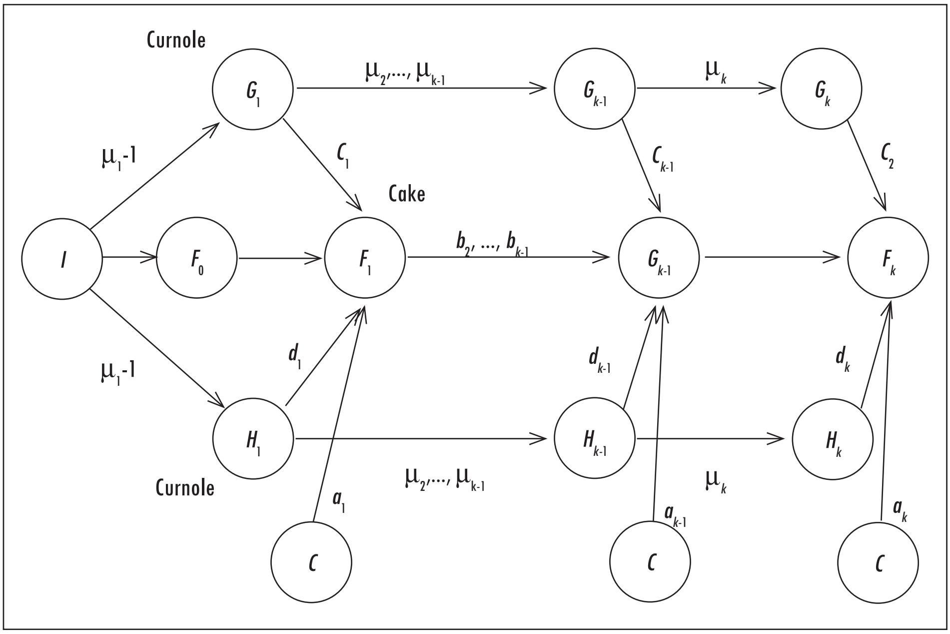
There is one linear layer with a unique neurone, F_0, which represents a current accumulated knowledge element or CAKE node. Define y_t as the time series $\{y_1,...,y_T\}$. The variable $X = \{X_t;\ t = P + 1,..., T\}$ is an input matrix where each row, $X_t = \{x_{tj};\ j = 1,...,m\}$, corresponds to the lagged values of the a variable explaining y_t. P is the maximum lag considered when we built X_t. The node $F_0(X_t)$ is defined as an autoregressive model, such that:

$$F_0(X_t) = a_0 + \sum_{j=1}^{m} b_{0j} x_{tj} \tag{1}$$

The next hidden layers are composed by four neurones: the first neurone has no input connections and its output always has unit value; this neurone is noted by the letter *C* in Figure 1. In each hidden nonlinear layer, DAN2 uses a nonlinear transformation based on the projection of over a predefined and fixed reference vector $R = \{r_j; j = 1,..., m$ to normalize the data. Here, m is the number of columns of the row vector X_t. For this, the angle, α_t, between each data vector X_t and the vector R is calculated. Without loss of generality, R can be defined as a vector of ones as suggested by Ghiassi and Saidane (2005). Thus, the angle α_t is calculated as:

$$\alpha_t = \arccos\left(\frac{\sum_{j=1}^{m} x_{tj}}{\sqrt{m\sum_{j=1}^{m} x_{tj}^2}}\right) \tag{2}$$

Figure 1. The DAN2 Model



Source: (Ghiassi and Saidane, 2005).

Ghiassi and Saidane (2005) prove that this normalization (for the hidden layer k) can be represented by:

$$A \cos(\mu_k\ \alpha_t) + B \sin(\mu_k\ \alpha_t) \tag{3}$$

Which can be understood as a generalized Fourier series for function approximation. The constant μ_k is equivalent to a rotation and a translation of the reference vector R and it allows us to extract the nonlinear component in the data. The variation in the value of μ_k changes the projection of X_t over R and, as a consequence, its contribution to the final solution. Eq. (3) is represented in each hidden layer by two CURNOLE (current residual nonlinear element) nodes. The first CURNOLE node calculates the cosine function (the G_k nodes in Figure 1) and the second node calculates the sine function (the H_k nodes in Figure 1).

The output of each nonlinear hidden layer (and the output layer) is calculated in the CAKE (current accumulated knowledge element) node, F_k, as:

$$F_k(X_t) = a_k + b_k F_{k-1}(X_t) + c_k \cos(\mu_k \alpha_t) + d_k \sin(\mu_k \times \alpha_t) \tag{4}$$

Where α_k represents the weight associated to the C node; C_k and d_k are the weights associated to the CURNOLE nodes; $F_{k-1}(X_t)$ is the output of the previous layer, and it is weighted by b_k. Eq. (4) defines that the result of each layer is a weighted sum of the knowledge accumulated in the previous layer, $F_{k-1}(X_t)$, the nonlinear transformation of X_t (G_k and H_k nodes) and a constant (the C node).

1.2. Parameters Estimation for the Original DAN2 Model

The estimation of the model parameters is based on the minimization of an error measure, for example, the sum of squared errors:

$$SEE_k = \sum_t \left[y_t - F_k(X_t) \right]^2 \tag{5}$$

The optimization algorithm described by Ghiassi and Saidane (2005) is developed here using the following key points:

- The estimation process is sequential. At first, the parameters of the linear layer defined by Eq. (1) are calculated by ordinary least squares (OLS). While some stop criterion is not satisfied, a new layer is added to the model and fitted to the data. The parameters of previous layers remain fixed.
- DAN2 uses a layer-centered strategy and this means that only the parameters for the new added layer (a_k, b_k, c_k, d_k and μ_k) are optimized with the aim of minimizing Eq. (5).
- In Eq. (4), the sine and cosine functions repeat their values for $\mu_k < 0$ and for $(\mu_k \times \alpha_t) > 2\pi$, such that, it is only necessary to explore values of μ_k between 0 and max $(2\pi / \alpha_t)$.
- If μ_k is known, then the other four parameters for the current layer k in Eq. (4) are estimated using OLS; thus, the optimization problem is reduced to estimate the optimal value of μ_k.

Following the suggestions given by Ghiassi and Saidane (2005), we implement the following strategy for fitting the model to the data:

1. Let $k = 1$.
2. For the current layer k, take N points, $\mu_k(i)$, equally spaced over the range of μ_k, with $i = 1,\ldots, N$.
3. For each point in the grid, $\mu k(i)$, simultaneously calculate the parameters α_k, b_k, c_k and d_k using OLS and evaluate the fitting error using Eq. (5).
4. Keep the value of $\mu k(i)$ that minimizes Eq. (5).
5. While the error is higher than the minimum admissible error, add new layer to the model; let $k = k + 1$ and train the new layer following steps 2 to 3.

Note that in step 3 only one set of parameters a_k, b_k, c_k and d_k corresponding to the current layer k is estimated and the parameters of the previous layers $\{a_j,b_j,c_j,d_j;j = 1,\ldots,k - 1\}$ remain fixed.

2. The Modified DAN2 Model (mDAN2)

2.1. Proposed Architecture

In this section, we review the DAN2 formulae described by Ghiassi and Saidane (2005) and prove that the model can be written as an additive model, reducing the number of parameters to be estimated, and the required computational resources.

At first, we consider the DAN2 model with only one hidden layer. Thus, we replace Eq. (1) in Eq. (4) with $k = 1$:

$$F_1(X_t) = a_1 + b_1 F_0(X_t) + c_1 \cos(\mu_1 \alpha_t) + d_1 \sin(\mu_1 \alpha_t)$$

$$F_1(X_t) = a_1 + b_1\left(\alpha_0 + \sum_{j=1}^{m} b_{0j} x_{tj}\right) + c_1 \cos(\mu_1\alpha_t) + d_1 \sin(\mu_1\alpha_t)$$

$$F_1(X_t) = a_1 + a_0 b_1 + \left(\sum_{j=1}^{m} b_1 b_{0j} x_{tj}\right) + c_1 \cos(\mu_1\alpha_t) + d_1 \sin(\mu_1\alpha_t)$$

Regrouping terms and substituting a^* for $a_1 + a_0 b_1$ and b_j^* for $b_1 b_{0j}$, we obtain:

$$F_1(X_t) = a^* + \left(\sum_{j=1}^{m} b_j^* x_{tj}\right) + c_1 \cos(\mu_1\alpha_t) + d_1 \sin(\mu_1\alpha_t)$$

Which describes the output of layer 1 in terms of X_t. Repeating the described process for layers $k = 2,...,K$, it is easy to prove that the output for the last layer k is:

$$F_K(X_t) = a^* + \left(\sum_{j=1}^{m} b_j^* x_{tj} \right) + \sum_{k=1}^{K} \left[c_k^* \cos(\mu_k \alpha_t) + d_k^* \sin(\mu_k \alpha_t) \right] \tag{6}$$

In this form, we show that DAN2 can be formulated as an additive model. There are several advantages of using Eq. (6):

The model is lineal for the parameters a^*, $b_1^*, \ldots, b_m^*$, $c_1^*, \ldots, c_K^*$, $d_1^*, \ldots, d_K^*$ and nonlinear for μ_k, such that, when the values $\mu_1, \ldots, \mu_k$ are known, then the remaining parameters (a^*, $b_1^*, \ldots, b_m^*$, $c_1^*, \ldots, c_K^*$, $d_1^*, \ldots, d_K^*$) can be estimated using OLS.

When a new hidden layer is added, we add only three new parameters for the layer instead of the five parameters in Ghiassi and Saidane (2005). As a consequence, the complexity of the estimation of the parameters is reduced. Thus, the final model has *2K* less parameters than the original DAN2 model.

2.2. Parameters Estimation for the Modified Architecture

We use a variation of the optimization algorithm described in a previous section:

1. Let $k = 1$.
2. For the current layer k, take N points, $\mu_k(i)$, equally spaced over the range of μ_k, with $i = 1, ..., N$.
3. For each point in the grid, $\mu_k(i)$, calculate the parameters a^*, $b_1^*, \ldots, b_m^*$, $c_1^*, \ldots, c_K^*$, $d_1^*, \ldots, d_K^*$ using OLS and evaluate the fitting error using Eq. (5).
4. Keep the value of $\mu_k(i)$ that minimizes the error in Eq. (5).
5. While the error is higher than the minimum admissible error, add new layer to the model; let $k = k + 1$ and calculate all linear parameters following steps 2 to 4.

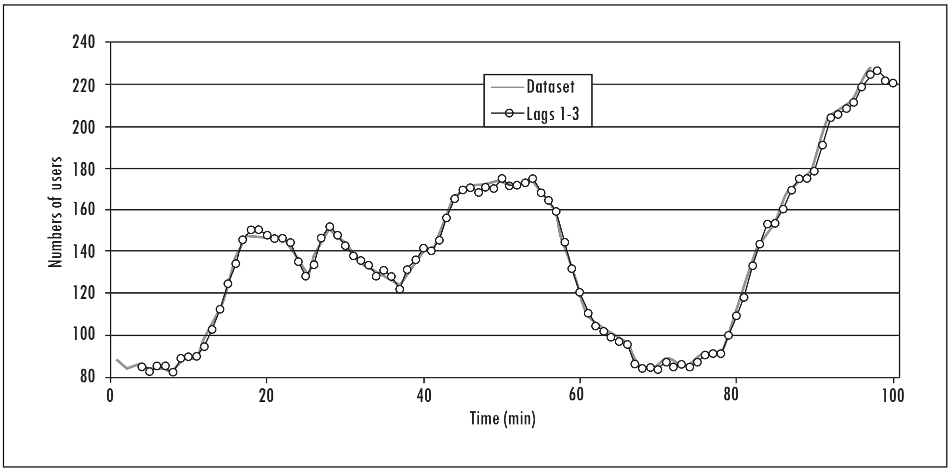
Note that the main difference with the original algorithm is the estimation of linear parameters. In the modified algorithm, when a new layer k is added, parameters a^*, $b_1^*, \ldots, b_m^*$, $c_1^*, \ldots, c_K^*$, $d_1^*, \ldots, d_{K-1}^*$ are estimated again along with the parameters for the current layer k, while the μ_k remain fixed, accumulating knowledge.

3. Numerical Experiment

The internet dataset (Ghiassi, Saidane and Zimbra, 2005) was used to test the quality of our approach (see Figure 2). This dataset has a length of 100 observations and it consists in the number of users logged onto an Internet server each minute. The first 80 points are used for fitting the model and the remaining 20 points for forecasting.

Ghiassi *et al* (2005) use this dataset for comparing the training and forecasting accuracy of DAN2 against an ARIMA(3,1,0) and a artificial neural network (ANN). For this, two DAN2 models were fitted: the former uses the lags 1–3 as inputs while the second uses the lags 1-4. In Table 1, we reproduce the mean squared error (MSE) and the mean absolute deviate (MAD) reported by Ghiassi *et al* (2005) for the four models.

Figure 2. Forecast One Period Ahead for the Internet Dataset Using the mDAN2 Model with Lags 1–3 and K=11



Source: Author's elaboration.

We tried to reproduce the errors reported by Ghiassi et al. (2005), but using our own implementation of DAN2. It was impossible to reach equal or lower errors than the reported by Ghiassi et al. (2005) for this time series. We agree with Guresen, Kayakutlu and Daim (2011) when they claim that some parts of the architecture of DAN2 are not clear enough, specially, the fitting algorithms. In a private communication with professor Ghiassi we inquired about the details of the training algorithm; kindly, the professor responded that there is not a free implementation of the algorithm because there is a project for implementing a commercial version of DAN2.

Table 1. Comparison of Results for the Internet Data Set

Model	Lags	Training MSE (MAD)	Forecasting MSE (MAD)
ARIMA (3,1,0) (Ghiassi et al., 2005)	1-4	9.76 (2.42)	8.11 (2.23)
Artificial neural network (Ghiassi et al., 2005)	1-4	7.00 (2.10)	9.25 (2.25)
DAN2-1 (Ghiassi et al., 2005)	1-3	1.81 (0.92)	4.15 (1.58)
DAN2-2 (Ghiassi et al., 2005)	1-4	2.78 (1.81)	3.87 (1.66)
DAN2-3 with $K = 20$ (this study)	1-3	4.06 (1.72)	5.09 (1.78)
DAN2-4 with $K = 6$ (this study)	1-4	3.69 (1,59)	6.46 (2.00)
mDAN2-1 with $K = 11$ (this study)	1-3	3.39 (1.47)	4.05 (1.64)
mDAN2-2 with $K = 6$ (this study)	1-4	3.47 (1.47)	5.62 (1.90)

Source: Author's elaboration.

Thus, we trained the original DAN2 model as described in Section 1.2 with a maximum of 25 layers. In figures 3a and 3b, we plot the MSE for the fitting sample as a function of the number of considered layers (K). As is described by Ghiassi and Saidane (2005), the fitting error is reduced each time a new layer is added.

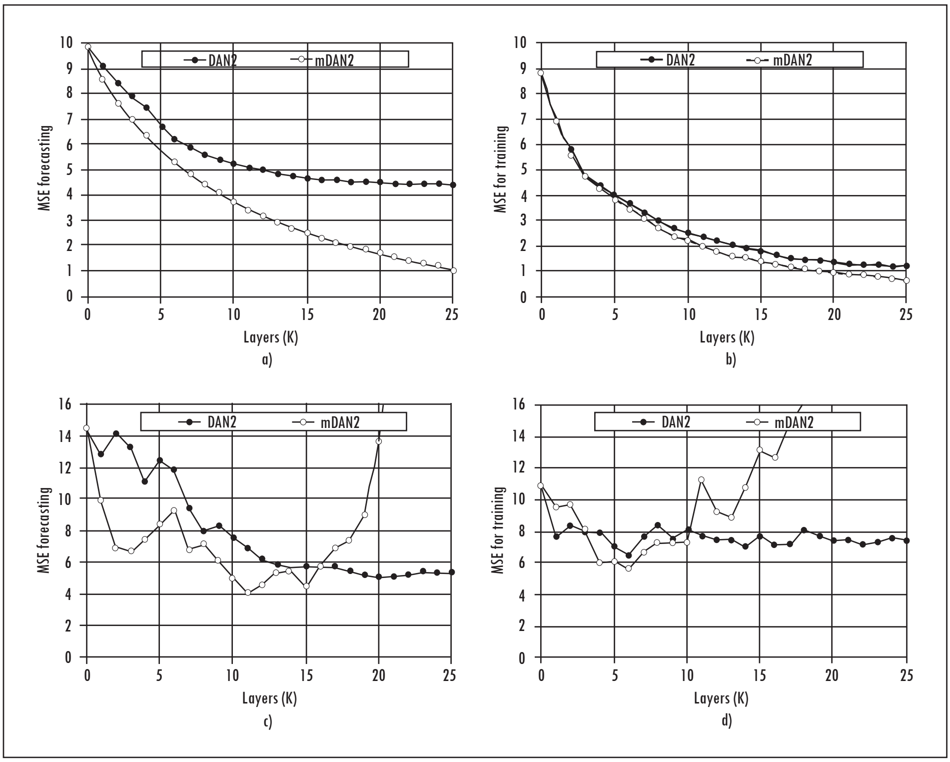
Also, we plot the MSE of DAN2 for the forecasting sample as a function of the number of layers (see Figures 3(c) and 3(d)). The lower forecasting errors are obtained for $K = 20$ and $K = 6$ when the lags 1–3 and 1–4 are used as inputs for the model respectively. The errors for the training and forecasting samples are reported in Table 1.

The mDAN2 model was fitted using the same implementation and parameters used for the DAN2 model as described in the previous paragraphs. In figures 3a and 3b. Also, we plot the MSE for the fitting sample as a function of the number of layers. For a low number of layers, the results are similar; but for a big number of layers the difference is high. In all cases (except for $k = 0$) the mDAN2 model has a lower fitting MSE than the DAN2 original model. Taking into account that we reduce in two the number of parameters of each layer ($2K$ in total), the mDAN2 is preferred in all cases by parsimony. Thus, we prove empirically that our specification is more effective for fitting the data.

Also, we plot the forecasting error for the mDAN2 model (see figures 3c and 3d); a simple inspection reveals that the mDAN2 has a better generalization that the DAN2 original model. Thus, we conclude (for the analysed case) that our representation of the original DAN2 model allow us to find models with a lower fitting error and better generalization; in addition, our model (mDAN2) has a lower number of parameters in comparison with the original version of

DAN2. From figures 3c and 3d, we prefer the models with $K = 11$ and $K = 6$ layers when the lags 1–3 and 1–4 are used as inputs for our neural network. The MSE and MAD for the fitting and forecasting samples are reported in Table 1. In comparison, the mDAN2 model has a better fitting and generalization that the implemented DAN2 model. Finally, we plot the one step ahead forecast for the mDAN2 with inputs 1–3 and $K = 11$ in Figure 2.

Figure 3. Fitting and forecasting MSE as a function of the number of layers. a) Fitting with lags 1–3; b) Fitting with lags 1–4; c) Forecasting with lags 1–3; d) Forecasting with lags 1–4



Source: Author's elaboration.

4. Conclusions

We prove that the "Dynamic Architecture for Artificial Neural Networks" (or DAN2) model can be formulated as an additive model and propose a modification to the original DAN2 estimation algorithm that simultaneously estimates all linear parameters using OLS. The proposed approach has several advantages;

first, it decreases the total number of parameters in the model in a factor of $2K$; second, all the linear parameters in the model can be estimated directly using OLS; and third, the optimization problem is reduced to obtaining an adequate set of values for . For validating the performance of our approach, we forecast one of the datasets used when the original DAN2 model was validated. We find that our modified DAN2 model performs better than the original DAN2 model using the internet dataset.

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