Seismicity and Hazard Evaluation of a Site[1]

Sarada K. Sarma

INTRODUCTION

In order to evaluate the seismic hazard of an engineering site we need the following information:

a) Historical seismicity of the region;
b) Geology and tectonics of the region;
c) A mathematical (statistical) model of analysis;
d) Local soil conditions at the site.

In general, the hazard analysis concerns with the first three factors while the local soil conditions are considered as a special case if necessary.

The study begins with the establishment of the region of interest around the site, which in general could be large, say 5° x 5° or even bigger. The idea is to establish regions within this area which can be called homogeneous in the seismic sense, i.e. that the region belongs to the same tectonic province, the earthquakes within the area has the same sort of mechanisms.

HISTORICAL SEISMICITY OF THE REGION

For the area in question, we then collect all the data about the earthquakes, i.e. the size (magnitude, moment), location (epicentre, focal depth) that has happened in the past. The data can be divided into two groups, instrumental data and pre-instrumental historical data.

The instrumental data can be obtained from the International Seismological Centre (ISC in UK), the National Earthquake Information Centre (NEIC in USA) and the National Geophysical Data Centre (NGDC in USA). These agencies can supply data covering the period from 1906 to the present. The accuracy associated with the instrumental data varies with time. At early stage, the errors associated could be large, particulary with epicentre determination (± 25 km). There are instances of gross errors

1 Resumen de la conferencia del doctor Sarada K. Sarma durante la conferencia internacional sobre dinámica de suelos organizada por la cátedra Jesús Emilio Ramírez del Instituto Geofísico de los Andes en mayo de 1997.

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in locations. The reason being very few instruments, unevenly located around the world and with low sensitivity. The location errors in the present time could be ± 5 km. In the early period, smaller events were not located and therefore incomplete.

The locations of pre-instrumental period earthquakes are obtained from historical studies, extending the period as far back as possible. Obviously, the historical earthquakes will concentrate on the large events. The magnitudes are determined from macro-seismic information, such as felt radius or epicentral intensity.

The magnitude determination for the instrumental period is non-homogeneous in the sense that different formulae were used in different periods. It is often necessary to recalculate magnitudes in a homogeneous way from the original data or look for published data. The error associated with the magnitudes could be of the order of ± 0.25.

The focal depth determination is not accurate at all. In the early period, the focal depth was generally given as "normal" or 33 km depth. In the catalogues, 33 km depth usually implies unknown shallow focus earthquake. Even in the present day, errors associated with focal depth determination could be large. Following the collection of this data, we have to map the epicentre locations, distinguishing between the instrumental and the historical ones and distinguishing the size.

GEOLOGICAL AND TECTONIC DATA

We map the known fault within the area, particulary the active faults, which moved in the quaternary period.

The combination of the two maps will give us an idea of the source region of earthquakes within the area. The sources therefore could appear to be points, lines (faults) and areas. The area source appears due to the uncertainty of the location of faults and the association of faults with epicentres. The source region determination is subjetive and not conclusive.

Activity of the source regions

It has been found that the activity of a source region follows a relationship (Guttenberg and Richter, 1954):

(1) $\log(N_c) = a - bM$

where M is the earthquake magnitude and N_C is the of earthquakes of magnitudes grater than or equal to M. In general, the numbers are normalized to a *year* and to *unit area* for area source and *unit length* for lineal source. a is therefore a measure of the activity of the region, when normalized. b is a measure of the 'brittleness' of the region. If the crust is highly faulted so that there exists many small faults and few large faults, then b will be large. There will be a tendency for many small earthquakes compared to large earthquakes. The value of b lies between 0.5 to 1.5. Considering the activity of the whole earth, b value is approximately equal to 1.

Due to the incompleteness of the data, deviation from the lineal trend exists. We generally do not consider magnitudes less than about 4 in the trend analysis. Also for the highest magnitude, since the period of the catalogue is very limited, this may have to be discarded in the trend analysis.

Maximum magnitude

For any region, we expect a maximum magnitude. It is essential to assess this maximum magnitude. From the study of the past earthquakes and the tectonic activity of the region, this can be estimated. In the absence of such a study, the largest historical earthquake plus a small increment (0.5) is generally considered.

STATISTICAL MODELS

The statistical model generally applied in hazard analysis is the Poisson process. The Poisson process is memoryless, which implies that earthquakes in one period of time does not depend on the past. This is therefore an assumption. However, it is acceptable for normal hazard analysis. When the hazard is controlled by the very large earthquakes, this assumption may lead to errors.

Return periods

the return of an event is simply the average time between events in the past and is given by the inverse of the annual frequency. If n is the number of *favorable events* per year, then the return period of the same is

(2) $T = 1/n$

Probability of exceedence

This is the probability of *at least one favorable event* in the life time of the structure. This is given by the expression

(3) $p = 1 - e^{-L/T}$

where:

p: probability of exceedence
L: life time of the structure
T: return period of the favorable event.

Attenuation model

To convert seismicity information to the ground motion, we need an attenuation model. This model should reflect the geology and the tectonics of the area. For example, the attenuation for intraplate earthquakes are different from that of interplate earthquakes. It is preferable to have attenuation relationship for the particular area concern. This relationship is the most important in the final result and should be chosen with care. Attenuation relationship for ground motion is of the general form:

(4) $y = b_1 e^{b2M} * r^{-b3} * e^{-b4r}$

With

(5) $r = \sqrt{d^2 + h^2}$

There are other forms of *r* as well such as

(6) $r = (d + c)$

Where b_i are constants dependent on regions.

Hazard evaluation

The point source model

This model is the basic "building block" for more elaborate source model such as a fault line source or an area source. In this model, a point source with an expected recurrence relationship (*a*,*b* parameters) is situated at a given distance *R* from the site and an attenuation relationship exist for the region. For the point source model, there are two approaches that can be adopted for the analysis.

- *Direct approach*: Given the expected life L of a structure and the acceptable probability of exceedance p, we can determinate the return period T of the event. Thus

(7) $p = 1 - e^{-L/T}$

The return period T is the inverse of the average number n of earthquakes per year.

(8) $T = 1/n$

n is related to the magnitude of the earthquake through the recurrence relationship

(9) $\log(n) = a - bM$[2]

(10) $y = b_1 * e^{b2M} * R^{-b3} * e^{-b4R3}$

Because of the presence of the maximum magnitude, this approach is applicable for a point source only.

- *Indirect approach*: This approach can be extended to more elaborate source models. This is a reverse procedure from the direct approach. We start with an assumed value of the ground motion y and determine its return period T which is then related to p.

(11) $y \Rightarrow M \Rightarrow n \Rightarrow T$

Many point sources model

In this model, for any given value of the ground motion y, the n values from all point sources are added together. The return period is then given by:

(12) $T = 1/ \sum n$

---

2 If the computed magnitude is bigger than the maximum magnitude, then M is the maximum magnitude.

3 In this relationship R is a distance parameter and not the distance directly.

ENGINEERING SEISMIC RISK ANALYSIS

Assumptions

Source

Earthquakes can happen anywhere within the "source region" with equal probability. Therefore, every point within the "source region" can be considered as independent sources.

Magnitude-frequency distribution

Number of earthquakes within the source region follow a magnitude frequency law. The cumulative and non-cumulative frequency distribution are as follows:

(13) $\log N_c = a - bm \qquad m_o < m \pounds m_u$

$N_c = 0 \qquad m > m_u$

(14) $\log \bar{N} = \bar{a} - \bar{b}m \qquad m_o < m \leq m_u$

$N = 0 \qquad m > m_u$

Where:

N_c: number of earthquakes of magnitudes $\geq m$ per unit area of source region per unit time.

N: number of earthquakes of magnitudes $m - \delta m/2 \leq m < m + \delta m/2$ per unit area of source region per unit time.

a, $\bar{a}$: define the *activity* of the source.

Relationships (13) and (14) are different. However, if m_u goes to infinity, then $b = \bar{b}$ and a can be derived from $\bar{a}$ and vice versa. Otherwise, if there is a maximum magnitude m_u, then use of either (13) or (14) will give slightly different results.

From (14) we can obtain:

(15) $n = \bar{N}/dm = \alpha^* \exp(-\beta m)$

Where:

(16) $\bar{\beta} = \bar{b} \ln (10)$

and

(17) $\bar{\alpha} = \bar{10}^a / dm$ $(\alpha = 10^a)$

(18) $N_c = \int_m^{mu} n dm$

$= - \bar{\alpha} / \bar{\beta} \{ e^{-\bar{\beta} mu} - e^{-\bar{\beta} m} \}$

$= \alpha \{ e^{-\beta m} - e^{-\beta mu} \} / \{ 1 - e^{-\beta mu} \}$

It is clear that unless m_u tends to infinity, $log (N_c)$ as derived from (14) in equation (18) is not a straight line and therefore not of the form (13). Which form to use in hazard analysis is dependent on data. Form (13) is more convenient . Form (14) may also be confusing due to the size δm of the histogram. On the other hand, form (14) is perhaps more natural. Difference in hazard using either of the two is small and appears at very small probability level and not of engineering consequence.

For $m = m_u$ equation (13) gives a finite value and equation (14) gives zero from equation (18)[4].

Arrival process

Earthquakes follow a Poisson arrival process i.e. that, within a time interval t, the probability p that there will be n earthquakes $\geq m$ is:

(19) $p = e^{-vt}(vt)^n / n!$

where

v: average number of earthquakes $\geq$ m (i.e. favorable events) per unit time ($= N_c$)

4 The lower bound magnitude m_o does not play any part in the numerical part of the hazard analysis. However, it is important in the engineering decision, so that the probability of the ground motion y corresponding to the lower bound magnitude occurring at the nearest point is a certainty.

The probability that "*at least one*" such earthquake will happen in the next year (any one year) is:

(20) $p = 1$ - probability that no such earthquake will happen

$= 1 - e^{-\nu}$

The probability that "*at least one*" such earthquake will happen in "*L*" years is:

(21) $p = 1 - e^{-nL}$

Attenuation relationship

The ground motion parameter (acceleration/velocity/displacement or any other) follow an attenuation relationship of the form:

(22) $y = b_1 e^{b2m} (R+c)^{-b3} e^{-b4R}$

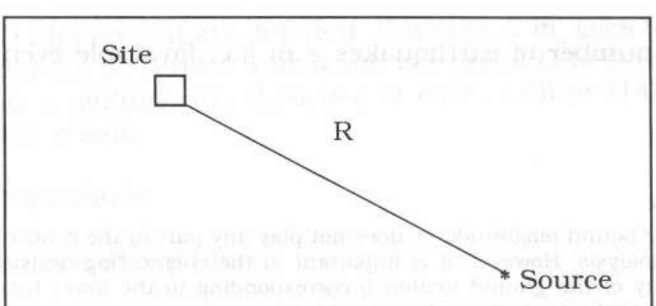
where b_1, b_2, b_3, b_4 and c are the constants of the relationship, R is the distance parameter and the magnitude parameter m is of the same kind as that in the magnitude frequency relationship.

Analysis

Point Source

In the case of a point source, the activity is defined in terms of unit time only. Obviously, unit area or unit length does not come into the picture. (see Figure 1).

Figure 1. One point source problem



We are interested in an acceleration (or any other parameter) $\geq y$ at the site. This acceleration $\geq y$ can be produced at the site by earthquakes of magnitudes $\geq m$ at the source, where, from the attenuation relationship:

(23) $m = 1/b_2 \ln \{y/b_1 (R+c)^{b3} e^{-b4/b2R}\}$

The average number ($n = N_c$) of earthquakes $\geq m$ at the source per year is

(24) $\nu = 10^a e^{-\beta m}$

$= 10^a [(y/b_1)^{-\beta/b2} (R+c)^{-\beta b3/b2} e^{-\beta b4/b2R}]$

Using the cumulate frequency distribution (Using the non-cumulate frequency distribution, a similar expression can be developed).

The probability that *at least one* earthquake of magnitude $\geq m$ and therefore producing acceleration $\geq y$ at the site will happen in year 1 is:

(25) $p_1 = 1 - e^{-n}$

$\approx \nu$ (for small values of ν)

With return period

(26) $T = 1/\nu = 1/p_1$

Finally, the probability that *at least one* earthquake of magnitude $\geq m$ and therefore produce acceleration $\geq y$ will happen in L years is:

(27) $p_L = 1 - e^{-\nu L} = 1 - e^{-L/T}$

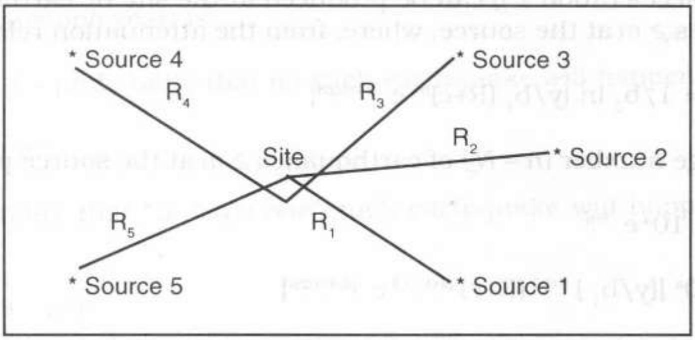
p_L is the probability of exceedance of the ground motion y in the life-time L of structure at the site[5].

S Independent point sources at different distances

Also, the attenuation constants may be different for different sources. (see Figure 2).

5 All solutions are valid only within the range of maximum magnitude *mu*. There is always a limit to *ymax* due to *mu*. For *b4 =0* and *c =0*, compare solution (26) with Cornell [1].

Figure 2. Independent point source problem



The first three steps of the point source problem can be worked out for each source, which gives m_i and n_i for each source. The probability that *at least one* earthquake of magnitude $\geq m$ and therefore producing acceleration $\geq y$ at the site will happen in year 1 is calculated as:

(28) p_1 = 1 - probability that *no* earthquake of magnitude $\geq m_i$ happen in 1 year in *any* of the sources.

$$= 1 - \prod_{i=1}^{S} e^{-n}$$

or

(29) $p_1 = 1 - e^{-\Sigma vi}$

$= \Sigma v_i$ (for small values of p_1)

With return period

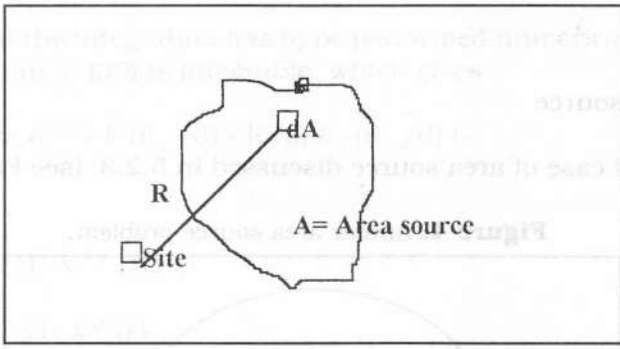
(30) $T = 1/\Sigma v_i = 1/p_i$

The probability that *at least one* earthquake of magnitude $\geq m$ and therefore produce acceleration $\geq y$ will happen in L years is the same as in point source problem (see equation (2)).

Area source

Als in Figure 3 the site can be inside or outside the source area.

Figure 3. Area source problem



An *area source* can be considered as *many point sources* of elemental area *dA* having the same *a* and *b* values and also having the same attenuation constants but at different distances. The activity is defined in terms of numbers per unit area per unit time. Therefore, for an elemental area source, the first two steps are the same for the point source.

The average number of earthquakes of magnitudes $\geq m_i$ at a distance R_i is:

(31) $$\nu_i = 10^a e^{-\beta m} dA$$
$$= 10^a \left[(y/b_1)^{-\beta/b2} (R_i+c)^{-\beta b3/b2} e^{-\beta b4/b2\, Ri} \right] dA$$

The probability that *at least one* earthquake of magnitude $\geq m$ and therefore producing acceleration $\geq y$ at the site will happen in year 1 is:

(32) $$p_1 = 1 - e^{-\Sigma \nu i} \text{ (considered as many point sources)}$$

$$\approx \Sigma \nu_i$$

$$\approx \int_A 10^a \left[(y/b_1)^{-\beta/b2} (R_i+c)^{-\beta b3/b2} e^{-\beta b4/b2\, Ri} \right] dA$$

with return period

(33) $$T = 1/I$$

where I, the integral in equation (32), can be written as:

(34) $$I = C^* G$$

where:

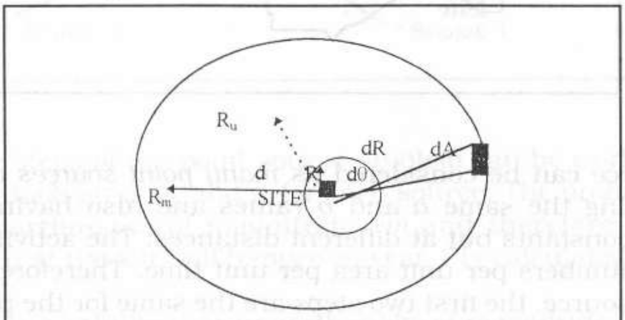
(35) $$C = 10^a (y/b_1)^{-\beta/b2}$$

(36) $G = \int_A 10^a [(R_i+c)^{-\beta b3/b2} e^{-\beta b4/b2Ri}]\, dA$ (Geometric function)

Anular area source

It is a special case of area source discussed in 5.2.3. (see Figure 4).

Figure 4. Anular area source problem



The site is at the centre of the circular area source of radius R_m and has a concentric small area around the site of radius *d* which is non-seismic. In this case, $dA = R\, dR\, dq$. R_u is the maximum distance, at which the maximum magnitude earthquake will produce the ground motion *y* according to the attenuation relationship and therefore, R_u is the solution of the equation:

(37) $(R_u+c)e^{b4\, Ru/b3} = (y/b_1)^{-1/b3}\, e^{-b2mu/b3}$

If R_u is bigger than the maximum source dimension R_m, then $R_u = R_m$

In this case:

(38) $G = \int_{\theta=0}^{2\pi} \int_{R=d}^{Ru} [(R+c)^{-\beta b3/b2} e^{-\beta b4/b2 R}]\, R\, dR\, d\theta$

(39) $= 2\pi \int_{R=d}^{Ru} [(R+c)^{-\beta b3/b2} e^{-\beta b4/b2 R}]\, R\, dR$

If the area source is only a sector of a circle, then replace 2π by the sector angle (in radians). The upper limit of the integral is either the maximum source dimension R_m or the upper limit of the distance R_u at which the maximum magnitude can produce the ground motion *y* whichever is smaller.

Special cases of the geometric function in equation (39)

If $b4 \neq 0$, then the integration has to be performed numerically only. If $b4 = 0$, then equation (39) is integrable, which gives

(40) $\bar{G} = 2\pi\, \bar{d}^{(1-\gamma)} \{ \bar{F}(\bar{R}_u/\bar{d}) - (c/\bar{d})\, F_1(\bar{R}_u/\bar{d}) \}$

where:

(41) $F(x) = [1 - x^{(1-\gamma)}]/(1-\gamma)$

(42) $F_1(x) = [1 - x^{-g}]/\gamma$

(43) $\bar{R}_u = R_u + c$

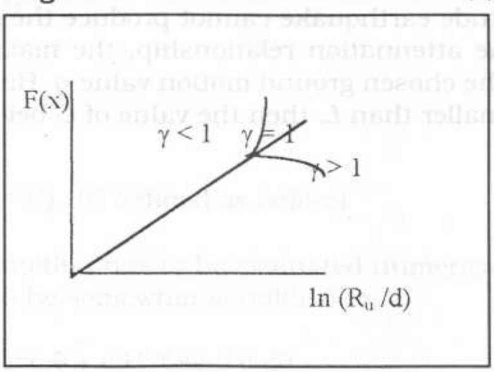
(44) $\bar{d} = d + c$

(45) $x = \bar{R}_u/\bar{d}$

(46) $\gamma = \beta b_3 / b_2 - 1$

The function *F(x)* is informative. It can be seen easily that (Figure 5)

Figure 5. The informative function F(x)



a) for $\gamma < 1$, the function diverges with increasing values of *x*.
b) for $\gamma = 1$, $F(x) = \ln(x)$ and therefore diverges with increasing *x*.
c) for $\gamma > 1$, *F(x)* approaches asyptotically to the value of $1/(g-1)$ for large *x*.

Similar statement can be derived for $F_1(x)$ where the critical γ value is 0. Therefore *F(x)* determines the convergence or divergence of the solution with increase *x*, where

(47) $x = R_u/d$.

If $F(x) = \infty$, then the probability $p = 1$ is a certainty for any value of y which is not meaningful. Therefore, in case that $\gamma \leq 1$, and *if magnitude is unbounded*, then a large source region around the site may give very large probabilities. This is unrealistic and the general cause of this problem is the unbounded magnitude. Therefore, the determination of the maximum magnitude is important, particularly If $\gamma \leq 1$[6].

Linear source-fault

In this case, the activity is represented as *numbers per unit length of fault per unit time.* A linear source can be considered as "many point sources" of elemental length *dl*, each having the same rate of activity and the attenuation relationship but at different distances *R*. The position of the fault with respect to the site is as shown in Figure 6 where:

L: Total length of the fault

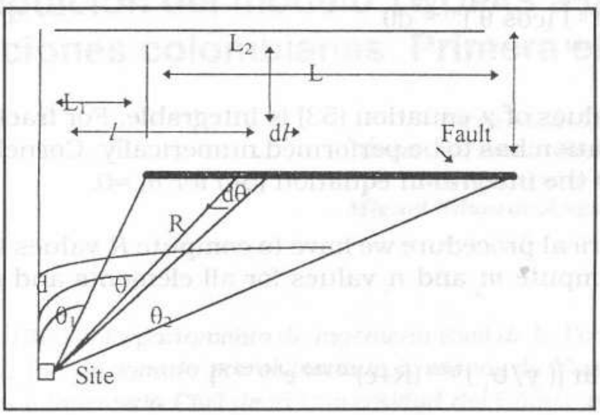
(48) $L = L_1 - L_2$

d: Normal distance of the fault from the site.

L_u: Effective length of the fault, outside of which the occurrence of the maximum magnitude earthquake cannot produce the ground motion *y*. L_u depends on the attenuation relationship, the maximum magnitude earthquake and the chosen ground motion value *y*. But L_u is not greater than L_2. If L_u is smaller than *L*, then the value of *G* below equals zero.

6 When the distance parameter *R* is defined as the slant distance, *R*= √(*r2+h2*) and the distances *d* and *Rm* of annular source refers to surface distance *r* from the site, then the solutions are directly applicable except that the limits of the integration should be changed to the slant distance from the site. We can therefore have completely submerged source at a depth *h* even when *c* = *0* in this case.

Figure 6. Linear source-fault problem



In this case, the geometric function, *G*, takes the form:

(49) $$G = \int_{L_1}^{L_2} \left[(R+c)^{-\beta b_3/b_2}\, e^{-\beta b_4/b_2 R}\right] dl$$

where:

(50) $$R = \sqrt{l^2+h^2}$$

with

Return period:

(51) $T = 1/(G * C)$ (*C* defined as before).

The geometric function has to be evaluated numerically. For $b_4 = 0$, the equation (49) can be somewhat simplified to :

(52) $$G = d^{-\gamma} \int_{\theta_1}^{\theta_2} (\sec\theta + \bar{c})^{-(1-\gamma)} \sec^2\theta \, d\theta$$

where $\bar{c} = c/d$ and $\gamma = \beta b_3 / b_2 - 1$.

For $c = 0$,

(53) $$G = d^{-\gamma} Q$$

where:

(54) $Q = d^{-g} \int_{\theta 1}^{\theta 2} (\cos \theta)^{-(1-g)} \, d\theta$

Fr integer values of γ, equation (53) is integrable. For fractional values of g, the integration has to be performed numerically. Cornell [1] gives a set of curves for the integral in equation (54) for $\theta_1 = 0$.

In the numerical procedure we have to compute R_i values for all elements and then compute m_i and n_i values for all elements and ground motion values y_i:

(55) $m_i = \ln [(y/b_1)^{1/b2} (R+c)^{b3/b2} e^{b4/b2 R}]$

and

(56) $v_i = 10^a e^{-\beta m} \Delta L \; ; \; v_i = 0 \text{ if } m_i > m_u$

Similar procedures can be developed for any area source. In the hazard analysis, the minimum magnitude of interest does not influence the hazard directly. However, it is of engineering interest that all structures must be designed for the certainty of the minimum magnitude earthquake occurring nearest to the site.

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