Comparison of PMU Placement Methods in Power Systems for Voltage Stability Monitoring[1]

Comparación de métodos de ubicación de unidades de medición fasorial en sistemas de potencia para monitoreo de estabilidad de tensión[2]

Sindy L. Ramírez-P.[3]
Carlos A. Lozano[4]

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[3] Ingeniera electricista, Universidad del Valle, Cali, Colombia. MSc en Ingeniería Eléctrica, Universidad del Valle. Estudiante de Doctorado en Ingeniería Eléctrica y Electrónica. Integrante del Grupo de Investigación en Alta Tensión (GRALTA), Universidad del Valle. E-mail: sindy.lorena.ramirez@correounivalle.edu.co

[4] Ingeniero electricista, Universidad del Valle, Cali, Colombia. MSc en Ingeniería Eléctrica, Universidad del Valle. PhD en Ingeniería Eléctrica, Universidad de Strathclyde, Escocia. Profesor asociado a la Escuela de Ingeniería Eléctrica y Electrónica, Universidad del Valle. Integrante del Grupo de Investigación en Alta Tensión (GRALTA), Universidad del Valle. E-mail: carlos.a.lozano@correounivalle.edu.co

Abstract

A power system is subject to events that may lead to voltage stability problems, which eventually may lead to black-outs. On-line phasor measurements may allow anticipating situations leading to voltage instabilities; these on-line measurements can be accomplished by using Phasor Measurement Units (PMU). However, for a power system it is is not economical or necessary to install PMU at each bus; in order to avoid this situation it is required to develop a strategy for PMU placement that is useful and convenient, with a limited number of available PMU. This paper presents a review and comparison of some methods for PMU placement in power systems; also, we include a classification according to the type of observability obtained and consider the application of the method . Finally, a method for monitoring voltage stability in power systems is chosen and tested on the IEEE39-bus system using Matlab-PSAT (Power System Analysis Toolbox).

Keywords

phasor measurement unit (PMU); voltage stability; contingency; optimal PMU placement; power system

Resumen

Un sistema de potencia está sujeto a eventos que pueden causar problemas de estabilidad de tensión, los cuales eventualmente pueden conducir a apagones. Las mediciones fasoriales en línea permiten anticipar situaciones que conducen a inestabilidad de tensión. Estas mediciones fasoriales en línea se logran usando unidades de medición fasorial (PMU). Sin embargo, para un sistema de potencia no es económico ni necesario la instalación de PMU en cada nodo. Para evitar esta situación se requiere desarrollar una estrategia de ubicación de PMU útil y conveniente, con un número limitado de PMU disponibles. Este artículo presenta una revisión y comparación de métodos para la ubicación de PMU en sistemas de potencia; además, se hace una clasificación de acuerdo con el tipo de observabilidad y se considera la aplicación del método. Finalmente, se selecciona un método para el monitoreo de estabilidad de tensión en sistemas de potencia y se realizan pruebas sobre el sistema IEEE39 nodos usando la herramienta Matlab-Power System Analysis Toolbox (PSAT).

Palabras clave

unidad de medición fasorial (PMU); estabilidad de tensión; contingencia; óptima ubicación de PMU; sistemas de potencia

Introduction

Phasor Measurement Units (PMU) installation in each bus of a power system is neither economical nor necessary. However, useful and convenient PMU location strategies with a limited number of available PMU must be developed. The PMU placement strategy is determined directly by the intended applications. The Electric Power Research Institute (EPRI) considers that the optimal placement of PMUs should be determined based on two main factors: The characteristics of the system and the PMU applications [1].

Researchers have developed topological or numerical methods to determine the optimal location of PMUs, which minimize the cost of installation in the system, while still achieving full observability with a minimum number of phasor measurements. These methods perform an optimization process taking into account all system buses.

An important consideration for the development of these PMU location techniques is determining the buses to be equipped with them, according to installation restrictions and the application of the data provided by these measuring devices.

Earlier reports dealing with PMU placement, such as [2]-[4], have classified the PMU placement problem according to the methods and algorithms that are used for analyzing the problem of choosing the optimal number and location of PMUs installation.

This paper provides an overview of the different placement methods for different applications reported in the literature. However, the selection of the specific method for determining the PMU placement will consider the voltage stability monitoring system, using the measurements provided by them with N-1 contingency in lines. The selected method is applied to the IEEE39-bus test system.

1. Optimal PMU Placement Methods

The minimal PMU placement is often termed as Optimal PMUs Placement (OPP). OPP refers to the minimum number of PMUs to be placed in the network

and while maintaining the ability to extract all the necessary data for a given application. In this context, OPP is a combinatorial problem i.e.: from a total set of K system's substations (or buses), N must be equipped with PMUs. N can be any number between 1 and K, which means that the number of combinations is given by:

$$number\ of\ combinations\ = \sum_{i=1}^{k} \binom{k}{i} = \sum_{i=1}^{k} \left(\frac{K!}{(K-i)! \cdot i!} \right) \quad (1)$$

This is a large number even for small systems. For example, the IEEE14-bus test system has approximately 16,000 possible combinations.

Many real systems have over a hundred buses, which makes it impossible to try all combinations for the best solution, so the minimal solution must be found in a different way. One type of solution methods used to solve this kind of problems is iterative search algorithms. Examples of such methods are: Tabu Search, and Simulated Annealing and Genetic Algorithms. A common characteristic to all of them is that none can guarantee finding the global minimum, but they can usually find a solution close to the minimum. OPP calculation is always subjected to a number of constraints. The most common constraint is the system full topological observability. This means that the necessary data from the network can be either directly measured or it can be indirectly calculated. Finally, the observability of topology constraint is something different from topological observability despite the similar names. In short, it means that it is possible to detect the topology status of the network. This topology can be changed if, for example, the power lines become disconnected due to contingencies.

Some review and classification works such as [5]-[7] dealing with the methods for placement of PMUs in power systems. We propose the following classification considering the purpose of application of data PMUs, the full observability of topology constraint, which is a condition required for estimation of state, and the voltage stability monitoring a power system.

1.1. Methods for Observability

A system is observable when its state variables can be determined from the set of available information. A concept for power system observability can be expressed as "a power system is observable if for a given topology and a set of available measurements, it is possible to determine the power flow across the system circuits" [8].

In [9], a heuristic algorithm for partitioning the power system into two or more subnets is proposed. This decomposition is performed by using a complete linear programming using vectors belonging to the spanning tree adjacent matrix from the power system network. Then, the PMU optimal placement is carried out to minimize installation cost. This paper considers the two approaches to observability: numerical and topological.

In [10], the authors present a theoretical and graphical method for PMU location based on incomplete observability, i.e., when the number of PMUs is not sufficient to determine the voltage of a power system (the electric network state). This methodology makes use of spanning tree graphics of the power system to find the PMU optimal placement based on a new concept that authors define as the "depth" of and incomplete observation. The method employs a simulated annealing algorithm to solve the PMU placement problem and the communication system. The essential contribution of this work lies in the systematic approach of the PMU placement across the network in stages, if necessary, so that the regions which have not been observed decrease gradually until the system is fully observable by the PMU. The concept of "depth" of incomplete observations ensures the uniform distribution of PMUs in the network, while limiting the distance between not observed buses and the ones being monitored.

The authors in [11] present a quick analysis method for power system topological observability. The method is based on the linearized power system state estimator model and uses augmented incidence matrix. The Optimal PMU placement (OPP) is focused on reducing the number of PMUs installed, subjecting the network to complete observability and sufficient redundancy. An optimization algorithm, Tabu Search, is proposed to solve the combinatorial optimization problem and includes a list of priorities based on heuristic rules to accelerate the optimization.

In [12] the authors present a method that seeks the optimal placement of synchronized phasor measurements, which are able to monitor voltage and current phasor along the branches of the network. Previous research about PMU placement have assumed that a PMU could be located at a bus and provide bus voltage phasor, as well as current phasor along all branches incident to the bus. The authors consider that the PMUs are designed to monitor a single branch by measuring the voltage and current phasor at one end of the monitored branch.

In [13] the authors introduce the concept of sensitivity-constrained optimal PMU placement. During the optimal PMU placement search for complete observability the sensitivity of the power system parameters is considered; in

addition, the most valuable dynamic data from power systems is obtained at the same time. The simulated annealing algorithm is adopted to find the minimal PMU placement for system observability. The objective is to minimize a discrete objective function with the restrictions considering that the system is topologically observable and the PMUs are placed at buses with higher sensitivities. The observability topology analysis method is used to calculate the sensitivities of power system buses. The initial estimate of the number of PMUs needed is provided by an observability topology analysis method, where the sub-graphic of measurements with PMUs is constructed from the data of the incidence matrix. PMU placement is done in the buses with the highest incidence branches and sensitivities in the non-observable region.

The authors in [14] present a location algorithm in order to have electrical system observability while also increasing the performance of secondary voltage control system scheme. The optimal placement problem (OPP) is formulated to minimize the number of PMU facilities subject to full network observability while the voltages of all buses in the system can be monitored in real time. The branch and bound optimization method is adopted to solve the OPP problem, which is suitable for problems with integer and boolean variables. Topology-based algorithm is used for observability analysis.

In [15] a two-stage method for the PMU placement is proposed; in the first step there is a minimum number of PMUs required to make the power system topologically observable, and the second step is proposed to check if the result of PMU placement (from step one) leads to a full ranked Jacobian measurement. In the event that the located PMU, ensuring topological observability in step one, does not lead to a full-rank Jacobian, a sequential elimination algorithm (SEA) in step two is proposed to find the optimal locations of additional PMU necessary for the system to be numerically observable.

In [16] the authors present a methodology based on a binary optimization of particle swarm (BPSO) for the optimal placement of phasor measurement units (PMU) when using a mixed measurement set. The optimal PMU placement problem is formulated to minimize the number of PMU installation, subject to full network observability and to maximize the measurement redundancy at the power system buses. In order to ensure full network observability, an algorithm based on the system topology is used, considering factors such as the available data from conventional measurements, the number and location of zero injection buses, the number and location of installed PMU and, of course, the system topology.

[17] provides an experimentation regarding the behavior of the PMUs in WAMS at their optimal locations. The scheme is based on comparing positive sequence voltage magnitudes of the entire network connected directly and indirectly to PMUs followed by positive sequence current phase differences for each interconnected line between different areas within the network. The purpose is to experimentally determine the ability of PMU to observe the entire network from its optimal location.

1.2. State Estimation Methods

The following methods for optimal placement of PMUs are based on the concept of static state estimation, formulated as a nonlinear set of equations, as follows:

$$z = h(x) + \in \tag{2}$$

Where:

z ($z \in R^m$):	Measurement vector
x ($x \in R^n$):	State vector
$\in$ ($E \in R^m$):	Measurement vector error
h ($h: R^m \rightarrow R^m$):	Relationship between measurement vector and state vector

Equation (2) is typically solved by the Newton-Raphson technique. The use of devices able to provide voltage and current phasors, such as PMU, produces a linear relationship between the state variables and the measurements of the variables, as follows:

$$z = Hx + \in \tag{3}$$

Where H ($H \in R^m \times ^n$) is the matrix of "state" of the system. Typically $m > n$, and the solution of equation (3) is obtained by the least squares method.

State estimation methods are intended to monitor the entire system with the minimum number of measuring devices, applying the concepts of linear state estimation and using the following placement general rules:

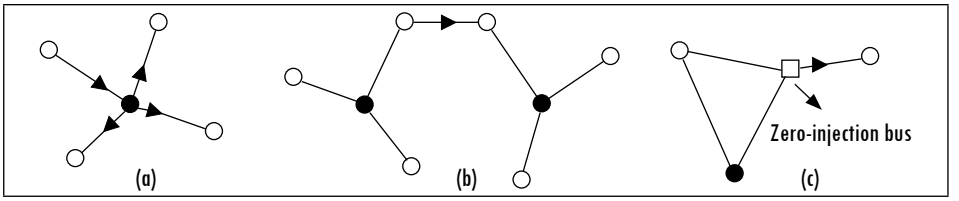
Rule 1: Assign the measurement to a bus where the PMU has been placed, including the measurement of current in each branch connected to that bus, see Figure 1a.

Rule 2: Assign a pseudo-measurement of voltage at each bus seen by a PMU.

Rule 3: Assign a pseudo-measurement of current to each branch bus connected to two voltage known buses seen in Figure 1b.

Rule 4: Assign a pseudo-current measurement to each branch where the current can be calculated indirectly using Kirchhoff's current law. This rule applies when the current balance in a bus is known. If the current N-1 bus incidents are known, the last current can be calculated by difference, Figure 1c.

Figure 1. PMU placement rules



Source: author's own elaboration

The Depth First method [18] uses only rules 1-3. The first PMU is located in the bus with the greatest number of branches connected. If there is more than one bus with these characteristics, it is chosen randomly. The following PMUs are placed on the same basis until completing the system observability

The Graph Theoretic Procedure [18] is similar to Depth First algorithm except it takes into account pure transit buses (Rule 4). The Recursive Security N Algorithm [19] is a modification of Depth First. The procedure can be subdivided into three main steps:

Generation of N minimum spanning trees: The algorithm is executed N times (N is the number of buses), using each network bus as the starting bus. Search of alternative patterns: At this point, the set of PMU obtained in the previous step are reprocessed so that each PMU is substituted in its connected bus with the bus that had a PMU installed in the previous step. The PMU placement, leading to a complete observability, are preserved. Reducing the PMU number in case of pure transit buses: in this step the network observability is still checked taking a PMU at once in each set. If the network has no pure transition buses, the procedure ends in the previous step. Finally, the sets of places having the minimum number of PMU are selected.

The Single-Shot Security N Algorithm is based only on topological rules and determines a single spanning tree [19]. The PMU minimum placement rules of the Recursive and Single-Shot Security N-1 Algorithms [19] assume a fixed network topology and a complete reliability of the measuring devices.

The criteria for a complete observability, in case of a line output (security N-1), considers the following rules:

Rule 1: A PMU is placed at the bus.

Rule 2: The bus is connected to at least two buses equipped with a PMU.

A bus is said to be observable if at least one of the above rules applies and this complete observability is ensured only if the line is lost and, generally, it is not enough to overcome PMU flaws. However, in most cases, with a limited number of additional PMU, either in some new buses or as redundant devices in the buses equipped by a PMU, a complete safety criterion N-1 can be achieved.

The procedure of the Recursive Security N-1 Algorithm starts from a bus and builds the spanning tree by assigning a PMU to the nearest bus, connected to the buses already observed. The procedure is repeated at every bus in the network and finally, it makes the selection of the minimum sets for PMU placement. The Single-Shot Security N-1 Algorithm is a variant of the Single-Shot Security N Algorithm and only differs in the criteria used to assign the PMU in the buses [19].

In [20] the authors propose a heuristic technique based on the condition of the minimum number of measurements from the measurement matrix. The authors take a simple technique for the measurement location method for the estimate of the state of the electrical system. The number of minimum conditions of the measurement matrix is used as a criterion in connection with the sequential removal to generalize the placement of the measurements. The method Singular Value Decomposition (SVD) is used to solve the state estimation. The algorithm provides a solution for the measurements placement of injection current and voltage that make the power system observable.

In [21] the uncertainty associated with the state variables of the power system obtained with the assistance of PMU is calculated. An integer-quadratic programming method is used to determine the minimum number and optimal PMU placement to ensure a full system topological observability. Three approaches are used to estimate the uncertainties in the state variables: the use of the theory of classical uncertainty propagation, the Monte Carlo method, and the random fuzzy variables (RFVs).

In [22] the authors present a method to obtain an overview of the power system condition, including state estimation, the methods needed to place the PMU and SCADA to offer the best properties of the state estimation problem -such as the network observability studied- the identification of erroneous data, and the accuracy of the obtained estimates. The authors suggest a genetic

algorithm (GA) for the PMU placement, which uses the following criteria: no critical measurements, maximum number of received measurements, estimates maximum accuracy, minimum cost of PMU installed, and transformation of the network graph into tree. The GA allows combining the location criteria above.

Graph theory plays a significant role in analysis methods due to its ability to represent the topological configuration of power systems. Thus, in [23] graph theory is used for the topological analysis of the placement problem by considering the number of available channels and branch outages.

Recently, the Gauss-Newton (GN) algorithm have been used in two studies by the same team [24], [25] to find the optimal PMU placement for a hybrid state estimation, by means of considering the accuracy and convergence of SE process. GN method is used in these studies to solve the iterative SE algorithm efficiently. Both studies use IEEE 30-bus and IEEE 118-bus to test the GN algorithm.

Thanks to the enhanced sensing capability, the PMU devices have been exploited in a gamut of power system monitoring tasks, ranging from line outage identification to state estimation. In [26] focuses on optimally selecting PMU locations for monitoring transmission line status across the wide-area grid. To bypass the combinatorial search involved, a linear programming reformulation is first developed to provide an upper bound estimate for the global optimum. Furthermore, a greedy heuristic method is adopted with only linear complexity in the number of PMUs, while leveraging on the upper bound estimate. A branch-and-bound algorithm is also developed to achieve a near-optimal performance at a reduced complexity.

A recent method is presented in [27] for the optimization of the cost of different parts of WAMS. In this method, the cost of optimal placement of phasor measurement units (PMUs) and a phasor data concentrator (PDC), as well as their associated communication infrastructure (CI), are simultaneously considered and minimized. For this purpose, the binary imperialistic competition algorithm is used for the optimal placement of PMUs. Dijkstra's single-source shortest-path algorithm is used to obtain the minimum CI cost. It is also used for optimal placement of PDC. In the proposed method, the optimal placement of PMUs and minimization of the cost of associated CI are carried out simultaneously. In other words, in addition to the optimal placement of PMUs, the optimal location of the PDC and minimal communication paths between PMUs and the PDC are obtained simultaneously. PMUs are located in such a manner that the network is fully observable in terms of the state estimation.

The method is implemented in different conditions of the power network such as N−1 contingency condition (e.g., a single-line outage or a single-PMU outage), and some important and practical considerations are provided. Practical considerations are the availability of preinstalled PMUs in some buses and the availability of the communication links in some parts of the power network.

1.3. Programming Based Methods

In [28] a simple algorithm for optimal PMU placement by means of the use of integer linear programming, which saves computing time, is presented. The contribution of this work is the proposed linear formulation, with and without conventional power flow and power injection measurements. Therefore, the solution to the optimal PMU placement problem is more efficient and can be used in practice. In [29] the author presents an extension of the proposed method including redundancy of the PMU placement. Due to the measurements of voltage and current phasor from PMU, accuracy, redundancy and, hence, the robustness of the state estimation is improved by integrating PMU measurements.

In [30] the authors present a method that focuses on the analysis of the ability to observe the network power flows and the PMU placement when using a mixed measurement set. The measurements and injections, as well as measurements of line voltage and current phasor, are provided by the PMU. The observability analysis is followed by a strategy of optimal PMU placement. The paper presents an integer programming formulation based on the solution associated with the PMU placement problem in power systems. The problem formulation considers the measurements as injections and power flows. The methodology finds a minimum number of PMU units with and without other conventional measurements. It also analyzes the case in which the network already has installed PMU and plans to place new units in the system for greater network observability.

1.4. Genetic Algorithms for PMU Placement

The authors in [31] consider the OPP through the simultaneous optimization of two objectives in "conflict" such as the reduction in the number of PMU and the redundant measurements maximization. They are coined "in conflict" since the improvement of one of them leads to deterioration of the other. So, in the paper, the authors choose to apply a so-called Pareto-optimal solution that involves the two objectives. A non-dominated sorting genetic algorithm (NSGA) is proposed for the PMU placement as a methodology for finding

these Pareto-optimal solutions. The Pareto-optimal concept can be explained in terms of a power ratio, i.e., for a multi-objective problem with an objective function that is desired to minimize simultaneously; a solution is said to dominate the other if it is better in at least one target. The algorithm is combined with a graph theoretical method and an easy genetic algorithm (GA) to reduce the initial number of candidate sites of PMU.

En [32] the authors investigated about the application of the immunity genetic algorithm (IGA) for the problem of PMU optimal placement in power systems. The problem is to determine the places for the location of a minimum number of PMU to make the system observable. The incorporation of an immune operator canonical genetic algorithm (GA), provided to retain the advantages of GA, uses some features and knowledge of the problems for the immobilization of degenerative phenomena during evolution, and improves the efficiency of the algorithm. This type of a priori knowledge about some parts of the optimal solution of the problem that exists in the PMU placement problem is deduced from the topological observability.

In [33] the authors present a new algorithm in order to determine the place and the minimum number of PMU to find the point of failure in power systems. The accuracy of the algorithm does not depend on the type of failure and resistance. The optimization problem is solved by means of a genetic algorithm (GA). This optimization algorithm has two advantages: first, it is economical; and second, it can be implemented in interconnected networks. The proposed algorithm is introduced in two steps. The first step consists in determining the optimal number of PMU and the installation places between all buses of the network. The second step is to determine the fault location by using the installed PMU measurements.

[34] employed the optimum location problem seeking to maximize the PMU redundancy and using the least amount of measurement equipment to ensure overall system observability using a swarm intelligence algorithm. This algorithm can solve the optimization problem, emulating the natural behavior of bees.

Recently, the research in [35] reused the Genetic algorithm as an intelligent technique for finding the optimal PMU placement for the state estimation of active distribution systems. In this research both the accuracy constraint the measurements deficiency are studied in addition to the effect of the DGs on the power flow. The Monte Carlo method is used to achieve optimal placement in addition to Genetic algorithm. Despite the cost of PMUs, more measuring devices and PMUs should be used to ensure the required accuracy for a robust SE.

1.5. Methods for Voltage Stability

In [36] a method that develops the Voltage Stability Load Index (VSLI) for a power system using data from PMU is presented. The optimal PMU placement was carried out considering islanding operating conditions. VSLI is estimated using a kind of recurrent neural network known as the Echo State Network (ESN). The development of ESN is computational efficient and provides accurate estimation. PMU placement for voltage stability monitoring is done in such a way as to ensure that the voltage phasor at all load buses are either direct measurements from PMU or calculated at first level of observability.

In our review we found different indexes for voltage stability monitoring, but these are based on the condition that for the calculation of the index PMUS must be installed and placed in the load buses. Some of these indexes are: Impedance Stability Index (ISI) [37], Voltage Stability Load Bus Index (VSLBI) [38], Voltage Stability Index (VSI) [39], Transmission Path Stability Index (TPSI) [40], Voltage Instability Predictor (VIP) [41], and Power Transfer Stability Index (PTSI) [42]. The aim of these techniques is to define a scalar that can be monitored as the system presents changes to different contingencies in order to allow operators and network analysts to perform the respective preventive and/or corrective action before a voltage collapse.

2. Review Analysis

The methods presented in the previous section, used to determine the OPP, are based on numerical observability or topological observability [43]. These two approaches have their own advantages and disadvantages. The approach based on numerical observability uses information (or gain) of the matrix or the Jacobian of measurement, which reflects the system configuration and the set of measurements. However, in the case of complex power systems the matrix can be very large and make the calculation slow. These techniques are iterative in nature, requiring a long time for convergence or the convergence will depend on the initial estimate.

Moreover, the methods based on the topological observability guarantee a complete network surveillance, but do not ensure a matrix of Jacobian measurements completely classified or organized [9]. However, a review of the literature reveals that in all the methods based on this concept, the PMU placement is conducted assuming that the power system is operating in steady state and, therefore, the optimal PMU placement cannot guarantee the observability of the entire system in case of a contingency in the electrical system. In [19], the authors

have considered the output of a single line as the PMU placement defined for complete observability.

The methods presented in the previous section were classified in Table 1 according to the type of observability. The difficulty in application of the method for the stability analysis is in the echo state network (ESN), the neural network training used for the estimation of VSLI. The neural network must be trained for each problem. It is also necessary to perform multiple tests to determine the appropriate architecture. The training is long and can consume considerable time. This does not make it attractive for use in different power systems.

Table 1. Classification according to the type of obse

Article reference	Type of observability
[9]	Numerical and topological
[10], [11], [12], [13], [14], [15], [16], [17], [18] (Depth First), [18] (Graph Theoretic Procedure), [18] (Bisecting Search Method), [19] (Recursive Security N Algorithm), [19] (Single Shot Security N Algorithm), [19] (Recursive and Single-Shot Security N-1 Algorithms), [21], [23], [31], [32]	Topological
[20], [22], [24], [25], [26], [27] [28], [30], [33], [34], [35], [36]	Numerical

Source: author's own elaboration

For monitoring voltage stability, the methodologies that consider the output of a line (contingency) in the power system and also satisfy the requirement of full system monitoring will be taken into account. To fulfill the purpose of monitoring, the method should be in the category of state estimation. The Recursive Security N-1 method and the Single-Shot Security N-1 method fulfilled these two conditions; the other methodologies only considered the system in steady state operation and in the case of a line output the complete system observability cannot be guaranteed. In fact, an optimal PMU placement should conduct a full state estimation also in case of changes and/or outputs of transmission system components.

These changes can be summarized as follows:

- Changes of injection buses, loss of generation, or load shedding.
- Changes of the branch admittance to zero in case of disconnection.
- Loss of a measuring device.

In case of variations in the power injection, the observability obtained by the N-1 security criterion is not lost, and the event could be detected even by measuring variations. Generation or load losses could be seen by improving the measuring redundancy of the bus with an additional pure transition bus. Moreover, changes in the network topology may lead to loss of observability in a certain area of the network. This may be a serious disadvantage when the state estimation is used for corrective actions, such as the voltage or transient stability assessment. Finally, when there is a failure of the PMU, the full network observability is also lost due to the construction of the minimum spanning tree and the associated inaccuracy depending on the specific device location.

3. Results

The two methodologies that meet the evaluation criteria (change of topology and complete observability of the system in stable state and under contingency N–1) were simulated using Matlab-PSAT version 2.1.5, developed by Professor Federico Milano of University College Dublin. This is an open source tool for analysis and control of power systems and it is freely distributed online [44]. The system chosen for evaluation is the IEEE39-bus system.

The methods evaluated are in the tool "PMU Placement" PSAT, which requires as input the results of the system load flow to estimate the state of the network. Finally, the simulation results of the two methods can be seen in Table 2. The simulation of the two methods here work with the system in steady state operation and with the output of one line at a time; therefore this makes the number of PMU to increase when compared to some results as in [9], [11], and [15] where the topology of the power system is considered fixed.

Table 2. Results of PMU location in IEEE39-bus sys

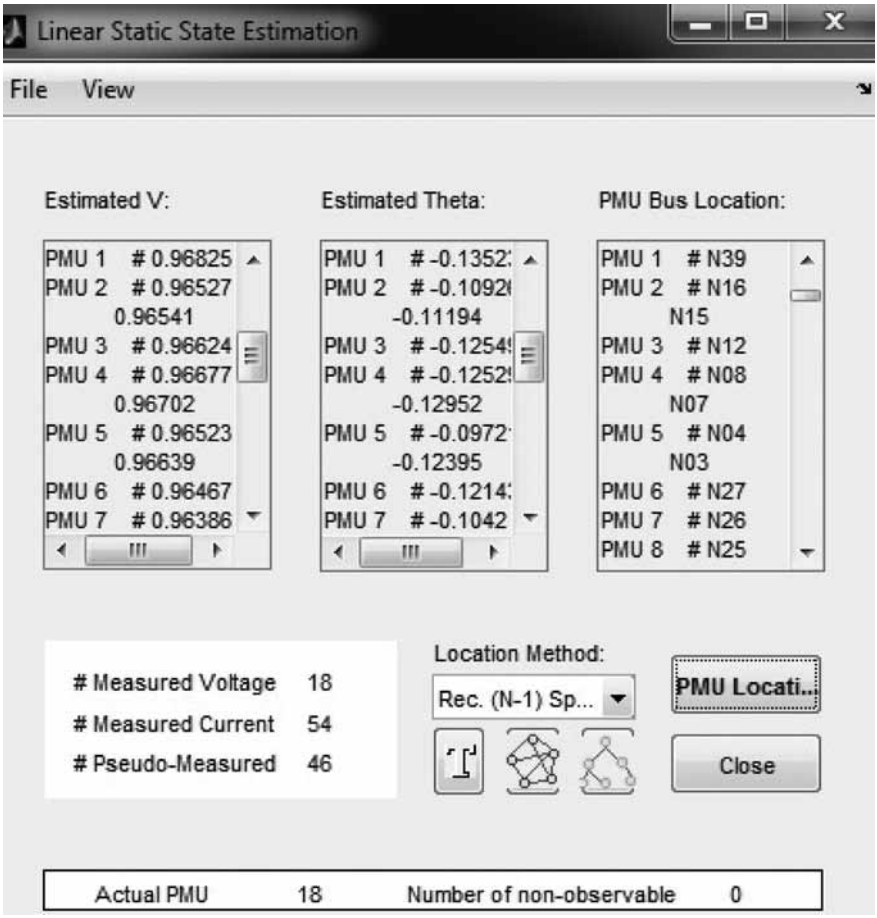
BUS	Recursive Security N-1 method						Single-Shot Security N-1 method
01	0	0	0	0	0	0	0
02	0	0	0	0	0	0	0
03	0	0	0	0	0	0	1
04	0	0	0	0	0	0	1
05	0	0	0	0	0	0	0
06	0	0	0	0	0	0	0
07	0	0	0	0	0	0	1

BUS	Recursive Security N-1 method						Single-Shot Security N-1 method
08	0	0	0	0	0	0	1
09	0	0	0	0	0	0	0
10	1	1	1	1	1	1	0
11	1	1	1	1	1	1	0
12	0	0	0	0	0	0	1
13	1	1	1	1	1	1	0
14	1	1	1	1	1	1	0
15	0	0	0	0	0	0	1
16	1	1	1	1	1	1	1
17	0	0	0	0	0	0	0
18	1	1	1	1	1	1	1
19	1	0	0	0	0	1	0
20	1	1	1	1	1	1	1
21	0	0	0	0	0	0	1
22	1	1	1	1	1	1	0
23	0	0	0	0	0	0	1
24	1	1	1	1	1	1	1
25	1	1	1	1	1	1	1
26	1	1	1	1	1	1	0
27	0	1	1	1	1	0	1
28	0	0	0	0	0	0	1
29	1	1	1	1	1	1	1
30	0	0	0	0	0	0	0
31	1	1	1	1	1	1	1
32	1	1	1	1	1	1	0
33	0	0	0	0	0	0	0
34	0	0	0	0	0	0	0
35	1	1	1	1	1	1	0
36	0	0	0	0	0	0	0
37	0	0	0	0	0	0	0
38	1	1	1	1	1	1	0
39	1	1	1	1	1	1	1
PMU	18						18
Set PMU	6						1
t(s)	3.0662						0.24262

Source: author's own elaboration

The method selected is the Single-Shot Security N-1 method which uses a set of PMU for IEEE39-bus system with a total number of 18 PMU located at the buses: 3, 4, 7, 8, 12, 15, 16, 18, 20, 21, 23, 24, 25, 27, 28, 29, 31, and 39 with a simulation time of 0.24262 seconds. This approach is chosen because there is only one set recommended for the placement; however the sets proposed for the Recursive Security N-1 method are six. This occurs because of the trees constructed by the algorithm to ensure observability, where each set has 18 PMU. Both methods include the concept of security N-1, i.e., the observability considers the output of one line, but not the failure of a PMU. Figure 2 shows the user graphical interface of the tool "PMU Placement" PSAT.

Figure 2. User graphical interface "PMU Placement" from PSAT



Source: author's own elaboration

4. Conclusions

In this work we conducted a comparison and analysis of the methods for PMU placement in power systems. An important factor in the placement method selection is the application intended to be considered for the PMU.

The interest for the PMU placement was the voltage stability monitoring; therefore the methods applied for state estimation, which aim to monitor the entire power system, were considered.

The voltage stability monitoring implies that the system works under different operating scenarios, including also the output of N-1 lines of the system; the Single-Shot Security N-1 method selected considers such contingencies that change the topology of the system.

Monitoring of voltage stability can be estimated from the system state, which aims to have a picture of the current state of the system. The data provided by the PMU can be used for the state estimation of the system, which simulate the start of PV curves. Thus, you have the advantage of working with accurate data entry as delivered by the PMUs for the analysis of state estimation.

It was found that methods that consider changes in the network topology like a line output, require a greater number of PMU; for full network observability it is required to install PMU in many of the buses in the system. This was verified by comparing the results obtained in other studies, which consider the system only working in steady state operation.

The method chosen for the location of PMUs is based on the theory of state estimation that aims at full or partial monitoring of the power system, and also has the advantage of not losing observability if there is any contingency.

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