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Global dynamics of humoral and cellular immune responses to virus infection

Appendix

I. To compute the basic reproductive number of the model (1) with immunity, we follow [14], [15] to form the next generation matrix, then

$$F=\begin{bmatrix} 0 & \beta_I \frac{k_s}{\mu_s} \\ N_1(\mu_I+\mu_s) & -N_2\beta_I \frac{k_s}{\mu_s} \end{bmatrix}, \quad V=\begin{bmatrix} \alpha_I \frac{k_T}{\mu_T}+(\mu_I+\mu_s) & 0 \\ 0 & \alpha_v \frac{k_B}{\mu_B}+\mu_v \end{bmatrix},$$

$$FV^{-1}=\begin{bmatrix} 0 & \beta_I \frac{k_s}{(\alpha_v \frac{k_B}{\mu_B}+\mu_v)\mu_s} \\ N_1 \frac{(\mu_I+\mu_s)}{\alpha_I \frac{k_T}{\mu_T}+(\mu_I+\mu_s)} & N_2\beta_I \frac{k_s}{(\alpha_v \frac{k_B}{\mu_B}+\mu_v)\mu_s} \end{bmatrix}.$$

From the last matrix the corresponding characteristic polynomial is

$$\lambda^2-\left(-N_2\beta_I \frac{k_s}{(\alpha_v \frac{k_B}{\mu_B}+\mu_v)\mu_s}\right)\lambda-N_1\frac{(\mu_I+\mu_s)}{\alpha_I \frac{k_T}{\mu_T}+(\mu_I+\mu_s)}\beta_I \frac{k_s}{(\alpha_v \frac{k_B}{\mu_B}+\mu_v)\mu_s}.$$

We cannot use [15], but we can use an extended result, which appears in Part II of this Appendix. Then we have

$$R_0=\frac{\beta_I\left[\frac{(\mu_s+\mu_I)}{\left(\alpha_I \frac{k_T}{\mu_T}+\mu_s+\mu_I\right)}N_1-N_2\right]k_s}{\left(\alpha_v \frac{k_B}{\mu_B}+\mu_v\right)\mu_s}. \tag{35}$$

Similarly, if the humoral and cellular immune response is not present, the basic reproductive number of the model (3) is

$$R_{0w}=\frac{\beta_I[N_1-N_2]k_s}{\mu_v\mu_s}. \tag{36}$$

The only difference with the previous number R_0 is that the immune activity is not present in the control of free viral particles and infected cells.

II.

Conjecture 12.1. *Let the characteristic polynomial of order n corresponding to the next generation matrix* FV^{-1} *be written as*

$$\Lambda(\lambda) = \lambda^n - a_{n-1}\lambda^{n-1} - \cdots - a_1\lambda - a_0,$$

with $a_i \geq 0$, $i = 0, \ldots, k$ *and* $a_j \leq 0$, $j = k+1, \ldots, n-1$. *Let* R_0 *denote the spectral radius of the next generation matrix, that is,* $R_0 = \rho(FV^{-1})$ *and*

$$R_0^* = a_0 + \cdots + a_{n-1}.$$

Then R_0^* *is a threshold value for the disease to take off or to die out in the sense that:*

1. $R_0^* < 1$ *if and only if* $R_0 < 1$.
2. $R_0^* = 1$ *if and only if* $R_0 = 1$.
3. $R_0^* > 1$ *if and only if* $R_0 > 1$.

Proof. It is the same proof as the one of Conjecture 1 in [15, p. 103]. □