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Control strategies in the spatial population dynamics of *Aedes aegypti* vector using sterile mosquitoes and insecticides

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Abstract

In this work, we analyze the population dynamics of the *Aedes aegypti* mosquito, a transmitter of various viruses such as dengue, zika, and chikungunya, in a given area, based on the use of various control techniques. To do this, we use a reaction-diffusion model that considers various environmental characteristics such as temperature and landscape shape. Initially, we do this analysis using larvicides, insecticides, and the SIT (sterile insect release) technique separately.

We simulate different control scenarios using appropriate numerical methods, test combinations of these techniques, and determine the efficiency of each strategy according to the overall reduction in the number of fertilized females due to the applied technique. Subsequently, through a cost-effectiveness analysis, we verified that the release of sterile mosquitoes at the beginning of each seasonal period is the best strategy to control the population of *Aedes aegypti*.

Keywords: *Aedes aegypti*; cost-effectiveness analysis; insecticide; larvicide; reaction-diffusion model; sterile mosquitoes.

1. Introduction

Today, there is a lot of information about *Aedes aegypti*, and it is know that this mosquito is the main insect responsible for the transmission of various vector-borne diseases, such as zika, dengue, and chikungunya. The epidemics and deaths that continue to cause these diseases show that the control capacity of this mosquito and, consequently, the reduction of diseases transmitted by this vector continues to be poor. Knowing its life cycle, its biology and its relationship with the environment has become a challenge for scientists, to the point of having a full knowledge that the climate and environmental conditions trigger its dynamics and consequently the spread of diseases; therefore, knowing about these characteristics is an important aspect for effective control.

There are many and varied models that study the control of mosquitoes using different methods that include or not control by SIT and analyze its population dynamics from theoretical and practical points of view, through systems of ordinary differential equations (ODE) just as in [1], [2], [3] and [4] and/or partial differential equations (PDE) involving dispersal, for instance, the early works of [5], the use of cellular automata by [6] and other like [7], [8], and [9]. All these studies have made a great contribution to understanding the dynamics of this mosquito in time and space. The model we present is an extension of the model studied in [10], where a numerical analysis of convergence was also carried out. This model differs from the previous ones, in that it was built specifically for the species *Aedes aegypti*, with the inclusion of spatial variables, parameter variation due to temperature changes, and a specific geographic context. The model describes the life cycle of the mosquito (aquatic and adult stage) and its particular characteristics are expressed by the parameters.

Vector control is commonly used in many parts of the world through the use of mechanical, chemical, or biological control methodologies, which may or may not be effective depending on many factors. In this work, we develop a mathematical model that allows us to estimate the population of the *Aedes aegypti* mosquito under certain environmental circumstances (landscape and temperature); we apply the traditional control strategies (larvicides and insecticides), and later we combine them with the sterile mosquito release technique (SIT); this technique consists of massive field releases of male mosquitoes sterilized by either irradiation or genetic modification during their mass-rearing. A mating between a wild female and a sterile male produces eggs that will not hatch; with a large number of these males, the crosses will not produce new mosquitoes, reducing the wild population [11]. Finally, we determine which of all these strategies provide the best cost-effectiveness benefit for the city of Cali, Colombia.

In [12], the authors estimated the entomological parameters of the vector *Aedes aegypti* at three points in the city of Cali: the Air Base (M. F. S.), the Airport (A. B. A.), and Universidad del Valle. Based on data presented in [13], they also deduced the polynomials used for the adjustment of some parameters with the specific temperatures of the Meléndez Campus of Universidad del Valle, during the years 2019-2020. The Meléndez Campus of the Universidad del Valle has specific characteristics related to the landscape, such as large areas covered with trees and grasslands and agglomerated areas of buildings, that greatly facilitate the application of our model. It is from these physical characteristics that we define the functions of the parameters that model mortality from larvicides and insecticides, consistent with the reality. In the same way, we delimit certain areas to define the variation of the parameter that models the dispersion.

This paper has the following structure. In **section 2**, we propose the compartmental model, together with the reaction-diffusion system and its parameters. In **section 3**, we characterize the application region and describe the entomological and control parameters, adjusted for our study region. In **section 4**, we describe the different strategies and, using appropriate numerical methods, present some simulations. In **section 5**, we make a cost-effectiveness analysis. Finally, in **section 7**, we present our conclusions.

2. Mathematical model

We developed a PDE reaction-diffusion model to study the population dynamics of *Aedes aegypti* mosquito when different control techniques are incorporated, such as the use of larvicides, insecticides, and genetically modified mosquitoes.

The model consists of six compartments: mosquitoes in the aquatic phase (A), mosquitoes in the adult phase, classified as wild males (M), females before mating or not fertilized (H), genetically modified mosquitoes or sterile males (N), females fertilized with viable eggs (F) and fertilized females with unviable eggs (G). The model incorporates the spatial dispersion of all populations except the aquatic phase. This type of model assumes that populations eventually disperse in all spatial directions and accounts for a mean movement of a random process [14].

The model considers the life cycle of the mosquito, into which sterile male mosquitoes are incorporated. A model of this form provides a SIT type model that was studied in [2] through an ODE system, for which the effect of these modified mosquitoes on the wild mosquito population is analyzed. In this model, in addition to spatial dispersion, we incorporate terms related to mortality from larvicides and insecticides; a parameter related to the failed sterilization of some mosquitoes, which could cause females to end up fertilized with viable eggs when they mate with these mosquitoes, and a parameter of sexual competence, which can be affected by the sterilization process, i.e., fitness or mating ability may be reduced. A compartment graph for the model is shown in **Figure 1**.

The oviposition rate *per capita* is given by $\phi(1 - A/K)$, where K is the environment carrying capacity and ϕ is the intrinsic oviposition rate. The population becomes adult with a rate γ, in proportion r of females and $1 - r$ of males.

Flows from H to F and G compartments depend mainly on the number of encounters of the female (before mating) with wild insects and modified males, and the corresponding mating fertilization rates. Here it is assumed that the probability of an encounter of a female with a wild insect is given by $M/(M + \lambda N)$ and with modified insect is given by $\lambda N/(M + \lambda N)$, where $0 \leq \lambda \leq 1$ accounts for the loss of sexual competition of sterile mosquitoes [3], [4]; note that $\lambda = 1$ indicates that there is no loss of fitness in the sterilized males. So, the *per capita* rate at which female insects are fertilized with viable eggs is

$$\beta \frac{M}{M + \lambda N} + \beta_N^* \frac{\lambda N}{M + \lambda N},$$

where β is the mating fertilization rate with wild insects and since sterilization may fail, β_N^* is the mating fertilization rate with failed modified insect. Here we have that $\beta_N^* = p\,(1 - q)\,\beta$, with $0 \leq p \leq 1$ the proportion of mosquitoes released on appropriate places, i.e. , places where they have a greater chance of mating with females, and $0 \leq q \leq 1$, the sterilization effectiveness of mosquitoes.

On the other hand, the *per capita* rate at which female insects are falsely fertilized (or fertilized with unviable eggs) is $\beta_N \lambda N/(M + \lambda N)$ where $\beta_N = pq\beta$ is the rate of fertilization by mating with successfully modified insects.

δ_n is assumed to be the number of sterilized mosquitoes released, μ_A, μ_M, μ_H, μ_N, μ_F and μ_G are the *per capita* natural mortality rates at each stage, and μ_I, μ_L are the mortality rates from larvicides and insecticides in the aquatic and adult stages, respectively. Finally, all populations except for aquatic stage A, move by diffusion, thus, α is the dispersion coefficient.

Populations A, M, H, F, G, and N, are all functions of space $\mathrm{x} = (x, y)$ and time $t \in (0, T]$, with spatial domain Ω an open and limited subset of $\mathbb{R}^2$ and smooth boundary almost everywhere $\partial\Omega$. Initial conditions A_0, M_0, H_0, F_0, G_0, and N_0 are properly defined in each situation of interest for each scenario; boundary conditions admit no flux through the boundary, and the latter implies that the derivatives of each population with respect to the outward normal vector η are equal to zero.

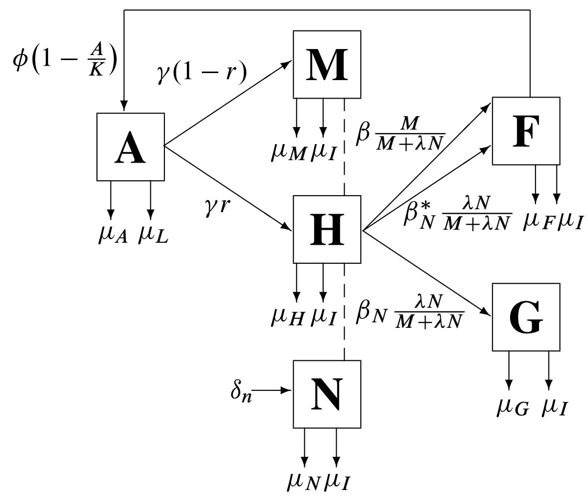


Figure 1. Compartmental graph that describes the flows of the population dynamics of the system in Equation 1

The reaction-diffusion PDE model that results from the above considerations is defined over the space-temporal domain $Q_T = (0, T] \times \Omega$ in the following form:

$$\begin{cases} \dfrac{\partial A}{\partial t} + (\mu_A + \mu_L) A = \phi \left(1 - \dfrac{A}{K}\right) F - \gamma A, \\ \dfrac{\partial M}{\partial t} - \nabla\cdot (\alpha_M \nabla M) + (\mu_M + \mu_I) M = \gamma (1-r) A, \\ \dfrac{\partial H}{\partial t} - \nabla\cdot (\alpha_H \nabla H) + (\mu_H + \mu_I) H = \gamma r A - \beta \dfrac{MH}{M+\lambda N} - \left(\beta_N + \beta_N^*\right) \dfrac{\lambda N H}{M+\lambda N}, \\ \dfrac{\partial F}{\partial t} - \nabla\cdot (\alpha_F \nabla F) + (\mu_F + \mu_I) F = \beta \dfrac{MH}{M+\lambda N} + \beta_N^* \dfrac{\lambda N H}{M+\lambda N}, \\ \dfrac{\partial G}{\partial t} - \nabla\cdot (\alpha_G \nabla G) + (\mu_G + \mu_I) G = \beta_N \dfrac{\lambda N H}{M+\lambda N}, \\ \dfrac{\partial N}{\partial t} - \nabla\cdot (\alpha_N \nabla N) + (\mu_N + \mu_I) N = 0, \\ \qquad t \in (0,T) - \{t_0 + n\tau,\ n = 0, 1, ..., p-1\},\ \mathrm{x} \in \Omega - \{\mathrm{x}_i, i \in S\}. \\ A(0,\mathrm{x}) = A_0,\ M(0,\mathrm{x}) = M_0,\ H(0,\mathrm{x}) = H_0, \\ F(0,\mathrm{x}) = F_0,\ G(0,\mathrm{x}) = G_0,\ N(0,\mathrm{x}) = N_0,\ \text{in } \Omega. \\ \dfrac{\partial A}{\partial \eta} = 0,\ \dfrac{\partial M}{\partial \eta} = 0,\ \dfrac{\partial H}{\partial \eta} = 0,\ \dfrac{\partial F}{\partial \eta} = 0,\ \dfrac{\partial G}{\partial \eta} = 0,\ \dfrac{\partial N}{\partial \eta} = 0,\ \text{in } (0,T) \times \partial\Omega. \end{cases} \tag{1}$$

Moreover, the equation for N satisfies the expression

$$N((t_0 + n\tau)^+, \mathrm{x}_i) = N((t_0 + n\tau)^-, \mathrm{x}_i) + \delta_n^i, \tag{2}$$

where $N((t_0 + n\tau)^{\pm}, \mathrm{x}_i)$ denotes the right and left limits of the function $N(t, \mathrm{x}_i)$ at time $t_0 + n\tau$, t_0 is the first day of release, $n = 0, 1, \ldots, p-1$, p the number of releases and τ the periodicity; $\mathrm{x}_i = (x_i, y_i)$ indicates the place where the modified mosquitoes are released and $i \in S$, the set of point indexes where the mosquitoes are released. In other terms, the system in Equation 1 evolves according to the equation for $\partial N/\partial t$ at all t and all Ω except at points and times of release, while N in the **Equation 2** is submitted to jumps at each point x_i and time $t_0 + n\tau$, accounting to the released sterile males [4], where δ_n^i is the number of released mosquitoes in the n-th release at a point i.

Note that the equation for G, corresponding to falsely fertilized females (or with unviable eggs) is decoupled from the system and does not play a role in the population dynamics; therefore, it can be ignored.

In [2], the temporal model associated with Equation 1 was studied, and its equilibrium points and the stability of each one were calculated and analyzed. In [10], an existence and uniqueness analysis was performed for a similar reaction-diffusion model (it did not include the β_N^* and λ parameters). This analysis was based on the study carried out by Bothe *et al.* in [15], and it was concluded that the problem proposed under certain conditions (hypotheses) on the data, has a unique global non-negative solution in the weak and classical sense.

3. Estimation of parameters

The Ω domain, on which we apply our model, corresponds specifically to Meléndez Campus of Universidad del Valle in the city of Cali (see **Figure 2**). It has an extension of $1.22\,\mathrm{km}^2$ of which a large part corresponds to green areas and open spaces. Within the Meléndez campus, a student population of approximately 5000 people transit per day.

In Figure 2, lines around the map of the campus delimit the total domain and the interior lines subdivide the domain into zones, whose characteristics concerning the landscape are different. The region identified as zone 1, corresponds to an area where a few buildings are located; zones 2 and 3 are mainly made up of green areas with trees; zone 4 corresponds to the main academic nucleus, where most of the buildings are located and where most of the people move; in zone 5 there is a lake and a few buildings with few people around; in zones 6 and 7, the sports center is located with an open sports fields and a pool. Each of these zones is characterized by the value of the dispersion parameter, which would represent to some extent environmental heterogeneity. Another aspect to take into account is that, some parameters are adjusted with functions that simulate mosquito elimination by larvicides and insecticides, considering how these chemicals work and are regularly applied in the city.

Our study is about a model that is applied to a specific region and therefore the parameters must be chosen in such a way that they adjust as best as possible to the conditions and particular characteristics of this region, i.e. , the parameters that we describe below are typical of the study region (if the model were applied to a different region, these parameters should be changed).

3.1. Dispersion coefficient by zones

Studies of the flight radius and spread of mosquitoes indicate that most females of *Aedes aegypti* can spend their entire lives inside or around the places where they have become adults, that they usually fly about 100-500 meters on average throughout their lives [16] and that only a few can go further when other factors (not considered in this study) allow it, such as travel in motor vehicles or when they are transported by people [17], [18]. The study of the dispersion of these vectors is of great interest for health, since variations in this parameter may be directly related to the effectiveness in the spread of different viruses. The active dispersal of the mosquito, especially

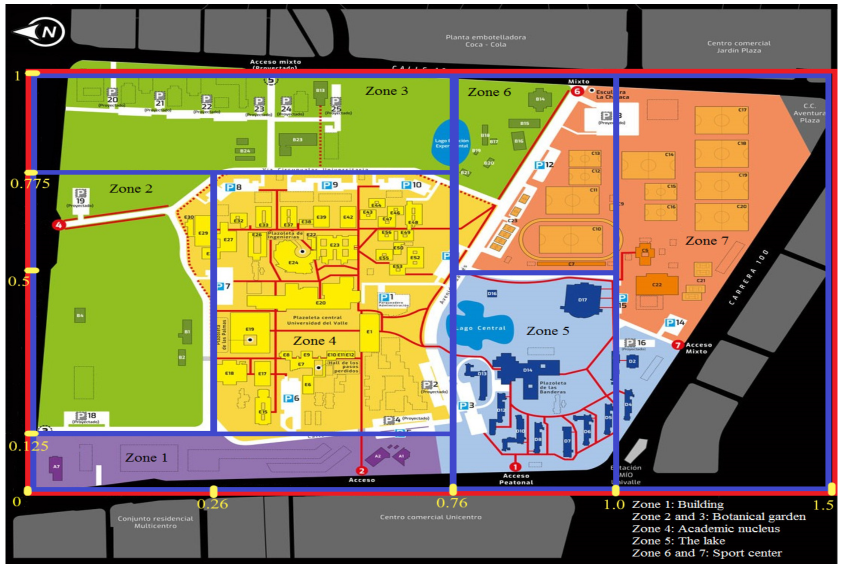


Figure 2. Study region: Meléndez Campus of the Universidad del Valle, its contour is delimited by red lines, and its different zones are demarcated by blue lines. Note the scale marked in yellow on the bottom horizontal line (positive x-axis) and the left vertical line (positive y-axis). Source: https://www.univalle.edu.co/la-universidad/mapa-del-campus

for females, responds to the search for blood sources to feed, resting places (shade and humidity), and places for oviposition. The dispersion term models this displacement of adult mosquitoes and also allows us to model landscape heterogeneity, which may mean, for example, the existence of factors or elements that could interfere with the displacement or dispersion of mosquitoes [19], [20]. Our study region has very well-marked physical characteristics, such as buildings or extensive grasslands, enabling us to divide it into 7 different zones (see Figure 2), and in each of them, we assume a value of the diffusion coefficient that best represents it.

If x_1, x_2,..., x_5 are the coordinates on the x axis of each vertical line (from left to right) and y_1, y_2,..., y_5, are the coordinates on the y axis of each horizontal line (from bottom to top) (see Figure 2), the dispersion coefficient α, can be written as the following constant piecewise function:

$$\alpha = \alpha_k(x, y) = \begin{cases} \alpha_1 & \text{if } x_1 \leq x < x_3 \text{ and } y_1 \leq y \leq y_2 \\ \alpha_2 & \text{if } x_1 \leq x < x_2 \text{ and } y_2 < y < y_4 \\ \alpha_3 & \text{if } x_1 \leq x < x_3 \text{ and } y_4 \leq y \leq y_5 \\ \alpha_4 & \text{if } x_2 \leq x < x_3 \text{ and } y_2 \leq y < y_4 \\ \alpha_5 & \text{if } x_3 \leq x < x_4 \text{ and } y_1 \leq y < y_3 \\ \alpha_6 & \text{if } x_3 \leq x < x_4 \text{ and } y_3 \leq y \leq y_5 \\ \alpha_7 & \text{if } x_4 \leq x \leq x_5 \text{ and } y_1 \leq y \leq y_5 \end{cases} \tag{3}$$

where $\alpha_1, \ldots, \alpha_7$ are the dispersion values for each zone described in Figure 2.

According to [21], the dispersion of the mosquito varies depending on where it is, on the street, inside the house, or inhomogeneous environments. This parameter will take higher values in those areas in which the mosquito does not stay for a long time, as they are areas that are not very suitable for its subsistence and where it can move easily, such as zones 2 and 3, and will take lower values in those areas where it can stay most of the time without having to travel much to find oviposition sites or food, because it is within areas suitable for its subsistence, such as zone 4 and 5. In the other zones, the dispersion takes intermediate values.

3.2. Temperature dependent entomological parameters

To estimate entomological parameters such as the rate of oviposition ϕ, maturation γ, mortality in the aquatic phase μ_A, and mortality in adult phases μ_M, the temperatures $\mathcal{T}$ of the city are taken into account. To make this adjustment, we used centered and re-scaled polynomials of degree m, which were estimated in [12] from linear least squares, applied to data experimentally collected by [13]; these polynomials have the form

$$P_m(\mathcal{T}) = b_0 + b_1\left(\frac{\mathcal{T}-\bar{\mathcal{T}}}{\sigma_{\mathcal{T}}}\right) + \cdots + b_m\left(\frac{\mathcal{T}-\bar{\mathcal{T}}}{\sigma_{\mathcal{T}}}\right)^m, \tag{4}$$

and minimize the error E given by

$$E = \sum_{j=1}^{\mathcal{N}} \frac{\left(P_m\left(\mathcal{T}_j\right)-\theta_j\right)^2}{\sigma_j}, \tag{5}$$

where m is the degree of the polynomial, $\bar{\mathcal{T}}$ and $\sigma_{\mathcal{T}}$ are respectively the mean and standard deviation of the temperatures used for the parameter measurements, $\mathcal{N}$ is the number of observations, θ_j is the estimated parameter for the temperature j and σ_j is the corresponding standard deviation. The coefficients b_i, the means $\bar{\mathcal{T}}$, and the standard deviation $\sigma_{\mathcal{T}}$ of these polynomials are given in [12].

On the other hand, due to the computational cost involved in the solution of the model (Equation 1), we will only consider nine of the twelve months of the year, i.e., we use a time horizon of 270 days, divided into three subperiods, every 90 days (3 months) and with different temperatures; subperiod of high temperature (ht) that goes from $t = 1$ to $t = 90$ corresponding to the months of June, July, and August; subperiod of medium temperature (mt) that goes from $t = 91$ to $t = 180$ for September, October and November; and subperiod of low temperatures (lt) that goes from $t = 181$ to $t = 270$ corresponding to December, January, and February. In each subperiod, the temperature is the same. These temperatures for the city of Cali were estimated using the data published on the website *www.weather.com*, which provides meteorological information for each city in the world, we take the daily temperatures of the city of Cali registered for the year 2019.

The temperatures of the city of Cali oscillate between 17 °C and 31 °C, without much variation from one month to another. There are two distinguishable climatic periods, the wet or rainy season and the hot or dry season. Based on this, in the first three months (June, July, and August), the temperature is relatively high; in the following three months (September, October, and November) the temperature is intermediate; and in the last three months (December, January, and February) the temperature is relatively low. The information collected only provides the data on the low and high temperatures of each day of the month, but it is not possible to establish which is the dominant temperature during the day. To estimate a fixed value of temperature, for each period, a weighted average is carried out as follows:

High-temperature period: A weight of 3 for the high temperature and a weight of 1 for the low temperature of each day.

Medium temperature period: A weight of 2 for the high and low temperatures of each day.

Low-temperature period: A weight of 1 for the high temperature and a weight of 3 for the low temperature of each day.

With these weighted averages for each month, a final average is calculated for the three months of each period and the standard deviation is calculated, resulting in the information in **Table 1**.

3.3. Induced mortality

All mosquitoes need water to complete their life cycle; however, not all prefer the same breeding sites, some prefer to lay eggs in small bodies of water as puddles or small containers, while others prefer large bodies of water as water sources. In the domain of application of our work, we approximately locate some points where these types of water bodies visibly exist, and we add some more points randomly since there may be other bodies of water that are difficult to see, and are not considered, these points are shown in **Figure 3**. In these specific sites or nodes, the carrying capacity K of the environment takes a maximum value, and, it takes the value of 1, which indicates that at that point there is no oviposition site and it will be this way throughout all 270 days. Larvicides are applied directly to these specific oviposition points and allowed to act until they lose their effect.

Several authors such as [22] have modeled the mortality rates induced by larvicides and insecticides from functions with exponential decay, assuming that the chemical persists in the environment for a certain period that will depend on climatic and environmental factors that are sometimes difficult to measure [17] therefore, measuring the residuality of each one is also a difficult and inaccurate process. However, despite the limitations it is possible to adjust them empirically despite the limitations. According to information provided by the Sanicontrol company, the residual effect of larvicides that are mainly applied in the city of Cali is approximately 5 days and that of insecticides is less than one day.

We model mortality from larvicides with a function that simulates how these are applied in the city of Cali, so we define the function

$$\mu_L(t) = \begin{cases} k_1 e^{-d(t-t_0)^2}, & t \geq t_0 \\ 0, & \text{otherwise} \end{cases} \tag{6}$$

where $d > 0$ accounts for the larvicide effect half-life $\sqrt{\ln(2)/d}$, time on which the larvicide effect is half of that at $t = t_0$), and $0 \leq k_1 \leq 1$ is the maximum larvicide effect.

Table 1. Means and standard deviations of temperatures in degree Celsius (°C) estimated for each subperiod.

Period	**Month**	**Mean (°C)**	**Standard deviation (°C)**	**Estimated temperature (°C)**
High temperature	June	28.1	0.9	28.6
	July	28.4	1.1	
	August	29.4	1.0	
Medium temperature	September	26.0	0.8	25.4
	October	25.1	0.9	
	November	25.1	1.8	
Low temperature	December	23.2	0.5	23.3
	January	23.1	1.3	
	February	23.6	0.5	

To define the function that models mortality from insecticides, we take into account that the domain is a rectangle and that in the fumigation process, the corners receive little or almost no insecticide because fumigation is applied mainly in the center, which is where the largest number of people and mosquitoes predominates. Among the many functions that can satisfy these characteristics, we choose the function

$$\mu_I(t,x,y) = \begin{cases} k_2 e^{-\left(a(2x-x_m)^2+b(2y-y_m)^2+c(t-t_0)^6\right)}, & t \geq t_0 \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

where $(x, y) \in \Omega$, $0 \leq k_2 \leq 1$ has the same meaning as k_1 in **Equation 6**; a, b are c are constants that characterize spatial and temporal residuality (insecticide effect half-life, $\sqrt[6]{\ln(2)/c}$; x_m and y_m are the maximum values for x and y coordinates; and t_0 is the application time.

We do not have scientific evidence that really supports the choice of these functions, but we believe that they are adequate as a first approximation to model the effect of these controls on the spatial and temporal domain under consideration.

3.4. Release of modified mosquitoes

The parameter δ_n^i in Equation 2 indicates the number of sterile mosquitoes released in a specific point i in the n-th release and let $\delta_n = \sum_i \delta_n^i$.

It is not easy to know how many modified mosquitoes should be released, it depends on many factors, [23]. We opted to determine this number based on the size of the population of wild male mosquitoes present at the time of release in the study area: 10 sterile males for each wild male [24], this counting process is done numerically. To describe the release methods we use Equation 2.

Method 1 comprises a single release at time $t = t_0$; i.e., $p = 1$ and the number of mosquitoes to be released is $\delta_0 = 10a_0$, where a_0 is the number of wild males present at time t_0 in the whole study area.

Method 2 includes ten releases, one per day; i.e. $p = 10$, $\tau = 1$ and $\delta_n = 10b_n$, $n = 0, 1, \ldots, 9$, where b_n is the number of wild males present at time $t_0 + n\tau$.

Method 3 is similar to method 2, but this time the releases are weekly, that is $p = 10$, $\tau = 7$ and $\delta_n = 10c_n$, $n = 0, 1, \ldots, 9$, where c_n is the number of wild males present at time $t_0 + n\tau$.

The methods we describe as method 4, method 5, and method 6 are similar to methods 1, 2, and 3 but this time the δ_n value will be a fixed value and equal to the maximum value used in each of these three methods, i.e., if

$$\delta_{m_1} = \delta_0, \quad \delta_{m_2} = 10\sum_n b_n \quad \text{and} \quad \delta_{m_3} = 10\sum_n c_n, \tag{8}$$

then

$$\delta_n = \frac{1}{p}\max\{\delta_{m_1}, \delta_{m_2}, \delta_{m_3}\}. \tag{9}$$

4. Strategies and numerical simulations

There are many and varied methods to solve complex systems of partial differential equations such as Equation 1; however, we prefer to use the classical scheme of the finite element method from linear elements in spatial variables [25], due to its good adaptability to complex geometries without loss of convergence, and the Crank-Nicholson scheme of the finite difference method for the temporal variable [26], because it is an unconditionally stable method, in turn, combined with a method predictor-corrector at each iteration in time for the treatment of nonlinearities [27]. Furthermore, while the coding was developing, convergence tests were performed for both methods to verify that the numerical procedure converges.

The numerical simulations were run in MATLAB software version R2020a. The spatial domain considered is a rectangle 1.5×1.0 kilometers long, that is $\Omega = [0, 1.5] \times [0, 1]$, divided into 60 and 40 subintervals in each direction, all of equal lengths, which generates a regular mesh with 2501 nodes and squares with 0.025 km on each side ($\Delta x = \Delta y = 0.025$). As our system is complex due to its numerical difficulties, we opted to simulate only 270 days of the year, which we consider sufficient time for the analysis of the strategies to be used; this period includes the months of June to February, a period with changes in the environmental characteristics of the city of Cali, Colombia, such as the variation of the temperature that affects mosquitoes life cycle. For the calculations, the time domain is the interval $[0, 270]$ divided into 5400 subintervals of length 0.05 days ($\Delta t = 0.05$).

We consider the seasonality for the entomological parameters ϕ, γ, μ_A and μ_i, $i = M, H, F, G, N$ for three different subperiods which were described in **subsection 3**. The values of the parameters for each subperiod are summarized in **Table 2** along with the variable parameters that shape the larvicide mortality (μ_L) defined by Equation 6, the insecticide mortality (μ_I) defined by **Equation 7**, and the number of sterile mosquitoes released (δ_n) defined by **Equation 9** for the methods 4, 5 and 6. The values for the dispersion coefficient α that were defines by a constant piecewise function in **Equation 3**, varies in the interval as shown in **Table 3**; these values, although they are assumed, are within the limits estimated by other authors, such as [21], keeping it always a little higher for the fertilized females, because they are expected to be the more agile in searching for blood to feed and for water to lay their eggs [28]. For the other parameters such as r, p, q, β, and λ, we assume the given values in Table 3.

Table 2. Estimated values of entomological parameters, functional values of variable parameters and number of mosquitoes released for each subperiod. Ht: High temperature, Mt: Medium temperature and Lt: Low temperature

Parameter	Description	Ht	Mt	Lt	Units
ϕ	Intrinsic oviposition rate	8.438	6.577	5.047	day^{-1}
γ	Conversion rate for adult	0.127	0.096	0.086	day^{-1}
μ_A	Natural mortality rate at aquatic stage	0.073	0.074	0.064	day^{-1}
μ_i	Natural mortality rate in each adult stage i	0.029	0.033	0.036	day^{-1}
μ_L	Mortality rate from larvicides	$[0, 1]$	$[0, 1]$	$[0, 1]$	day^{-1}
μ_I	Mortality rate from insecticides	$[0, 1]$	$[0, 1]$	$[0, 1]$	day^{-1}
δ_n	Number of sterilized mosquitoes released	876 139	238 048	85 510	mosquitoes

Table 3. Assumed values of constant parameters, equal for each temperature subperiod.

Parameter	Description	Value	Units
α_i	Dispersion coefficient in each zone i	$[0.0145, 0.0205]$	km^2/day
K	Carrying capacity of immature states	1 or 1000	eggs per breeding site
p	Mosquitoes released on appropriate places	1	dimensionless
r	Proportion of females	0.50	dimensionless
q	Fertilization effectiveness	0.90	dimensionless
β	Fertilization with wild insects	0.90	dimensionless
β_N	Fertilization with modified insects	0.81	dimensionless
β_N^*	Failed modified insect fertilization	0.09	dimensionless
λ	Loss of sexual competition	0.70	dimensionless

For the simulations, we consider breeding sites that are not uniformly distributed and are heterogeneous in size. Since mosquitoes preferably lay their eggs at breeding sites located close to ground level, the altitude is assumed to be homogeneous, as was done in [18]. We consider a total of 15 patches as shown in Figure 3 magnified over the domain in the form of small rectangles, each patch has the same value of carrying capacity K, and represents the sites where there are water sources generally used by mosquitoes to lay their eggs.

This distribution of breeding sites approximates in a certain sense the real location of some existing bodies of water within the Meléndez Campus, (for example, some fish ponds). In all the strategies presented, this distribution and the initial conditions are the same and are defined as follows: Variable A takes random values in the interval $[300, 1400]$ only over the 15 patches defined above and zero in the rest of the nodes, that is, A_0 is different from zero just in these 15 patches. For variables M and H, random initial values are defined for each node in the interval $[1, 40]$ in whole the domain, and the variables F, G, and N are all equal to zero also throughout the whole domain.

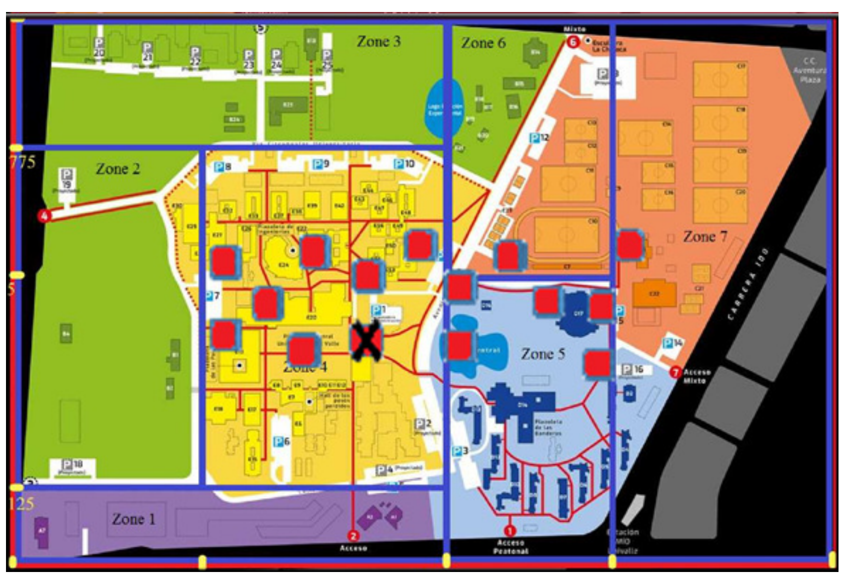


Figure 3. Distribution of breeding sites: Each red rectangle indicates the location of a breeding site on the domain. X indicates node for which some evolutionary graphs are plotted

To measure the efficiency of each control strategy, the behavior of the populations over time without controls is first simulated. Subsequently, each of the strategies described in the next section is simulated by applying different control techniques, such as larvicides, insecticides, sterile mosquitoes, and appropriate combinations of them. The efficiency of each strategy in the whole domain is measured as the percentage of decrease in the daily average of fertilized females due to the control strategy, and is measured from the expression

$$\sum = \left(\frac{F_s - F_c}{F_s}\right) 100\% = \left(1 - \frac{F_c}{F_s}\right) 100\%, \tag{10}$$

where F_c and F_s represent the total number of daily fertilized females across the domain during the whole period after using and without using a control strategy, respectively. That is,

$$F_c = \int_0^T F(t)dt \quad \text{and} \quad F_s = \int_0^T F(t)dt,$$

where $F(t)$ is given by the model using a control strategy and without control, respectively.

Scenario without control: **Figure 4** describes the scenario without control. We can see that the aquatic population keeps its relative distributions over time, as would be expected because the breeding sites are fixed over time, but the male and fertilized female populations change their initial distribution as time passes, concentrating the maximum number of mosquitoes in the center of the domain, where the breeding sites are located, and decreasing towards its borders. For all three populations, there is an increase in the high-temperature subperiod and a decrease afterward.

To have a better visualization of the effect of the controls applied in the scenarios of the following section, evolutionary graphs are plotted together,describing the behavior of the populations without and with control. For the aquatic population (A) and sterile males (N), it is only plotted at the node marked with an X in Figure 3, in which the initial conditions were defined different from zero; the reason for this is because the behavior is similar for the remaining 14 nodes, and for fertilized females (F) they are plotted for the whole domain.

4.1. Simulations applying chemical control

Three numerical simulations were carried out using larvicides and insecticides, which are described below.

Strategy applying larvicide – (L): Larvicides are applied directly on breeding sites and are left to act until they lose their effect. This application is carried out on days 0, 90, and 180, and mortality of this control is given by the function in Equation 6 with $k_1 = 1$ and $d = 0.03$, (values assumed so that the half-life of the larvicidal effect is equal to 5 days, approximately).

Strategy applying insecticide – (I): Insecticides are applied by thermoforming technique and only in the center of the domain. This control is applied five times for each seasonal subperiod, with a time interval of four days, i.e., on days 20, 24, 28, 32, 36, 110, 114, 118, 122, 126, 200, 204, 208, 212 and 216. The mortality for this case is given by the function in Equation 7, with $k_2 = 1$, $a = b = 1$, $y_m = 1$, $x_m = 1.5$ and $c = 6$, (values assumed so that the half-life of the insecticidal effect is equal to 17 hours, approximately).

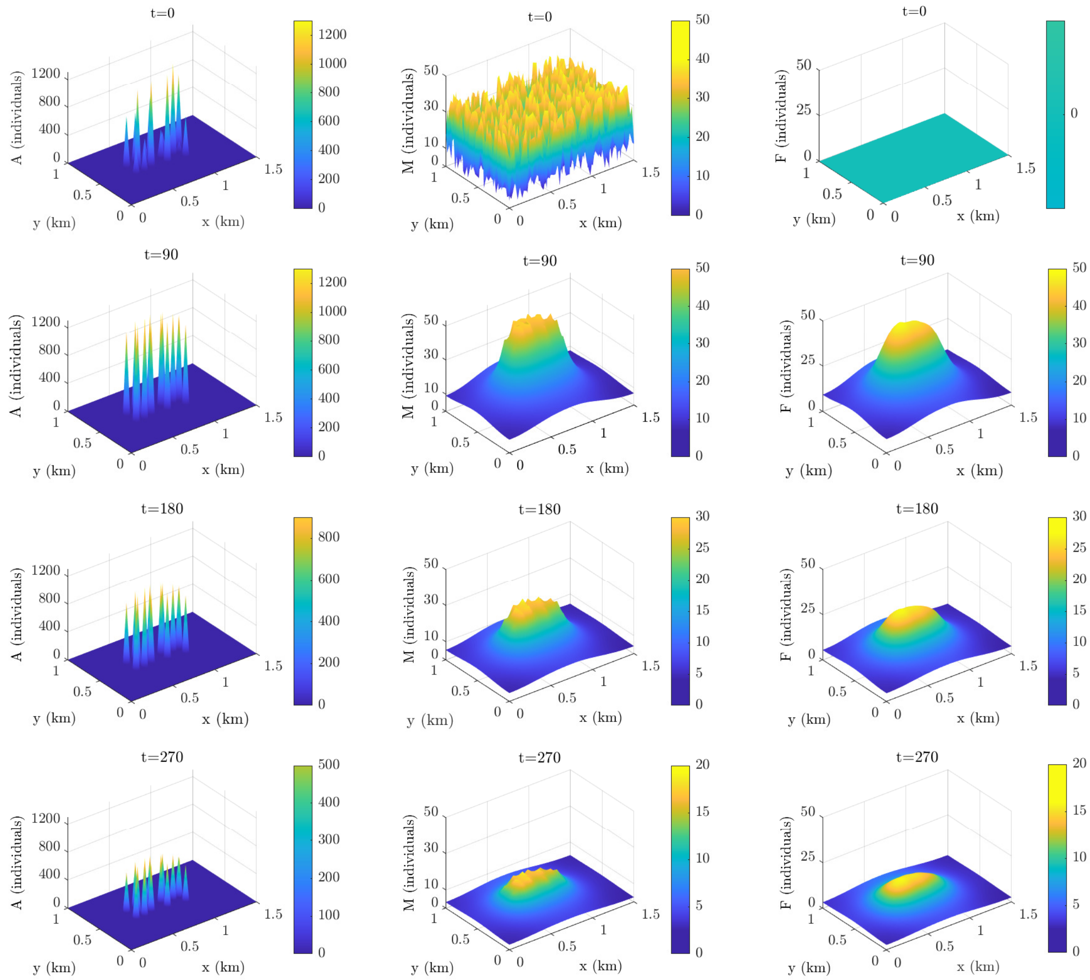


Figure 4. Scenario without control: Distribution of the populations in the aquatic phase (A), males (M) and fertilized females (F) over the domain, without control, at selected times

Strategy combining insecticides and larvicides – (IL): In this strategy, we apply the two controls described above. The insecticide is applied to the same moments and parameters of strategy (I), and the larvicide is applied on days 40, 130, and 220 with the same parameters of strategy (L)[1]. With this strategy, even though the spatial relative distributions are very similar, the decrease in the population of fertilized females is observed over the spatial domain as time passes (see **Figure 5**). In **Figure 6(a)**, note that the curves corresponding to the aquatic population without control $A_s(t)$ and with control $A_c(t)$ are plotted for a node where there is a breeding site. The curve $A_c(t)$ shows a considerable reduction of the population in the first days after applying larvicide and a small reduction during the days in which insecticide is applied. In **Figure 6(b)** the curves corresponding to the female populations without control $F_s(t)$ and with control $F_c(t)$ are plotted for the whole domain, show an increase in the first subperiod, but because the temperature drops during the second subperiod, which makes the entomological parameters change, the population decrease. This repeats during the third subperiod when the temperature drops again and the entomological parameters change once again. Also note that for each set of entomological parameters corresponding to the temperature of each subperiod, the populations (aquatic and

[1]Combining these strategies in the other order (LI) gives smaller effectiveness.

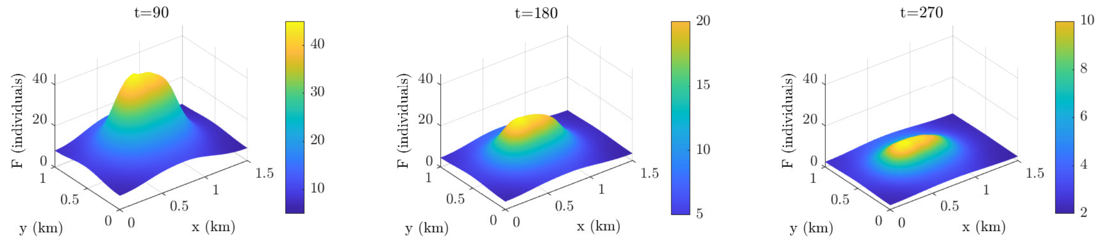


Figure 5. Strategy combining insecticides and larvicides (IL). Fertilized female distribution over domain at selected times

fertilized females) tend to stabilize at a value, which is less and less, and that the highest-profile of mosquitoes occurs mainly on almost a third of the whole domain, as can be seen in Figure 4. In **Figure 6(c)**, we can see that the effectiveness is much better in the first subperiod of time (high temperature).

4.2. Simulation using the sterile insect technique (SIT)

The following scenarios shows the population dynamics when the sterile insect technique is applied. In all cases, the released mosquitoes were evenly distributed in the set of nodes $S = \{607, 839, 1008, 1294, 1498, 1621\}$, which are close to the breeding sites, i.e., $\delta_n^i = \delta_n/6$ for $i \in S$, where δ_n was defined in **subsection 3**, and $\delta_n^i = 0$ for $i \notin S$.

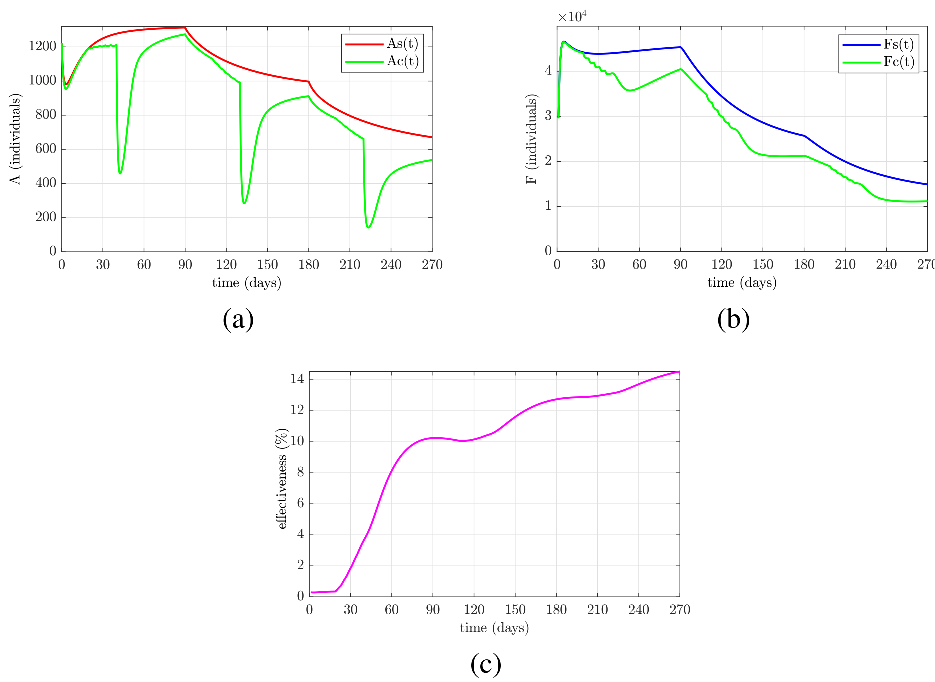


Figure 6. Strategy combining insecticides and larvicides (IL). (a) Aquatic population in node 1004, without control ($A_s(t)$) and with control ($A_c(t)$). (b) Fertilized females throughout the whole domain without control ($F_s(t)$) and with control ($F_c(t)$). (c) Effectiveness of the strategy.

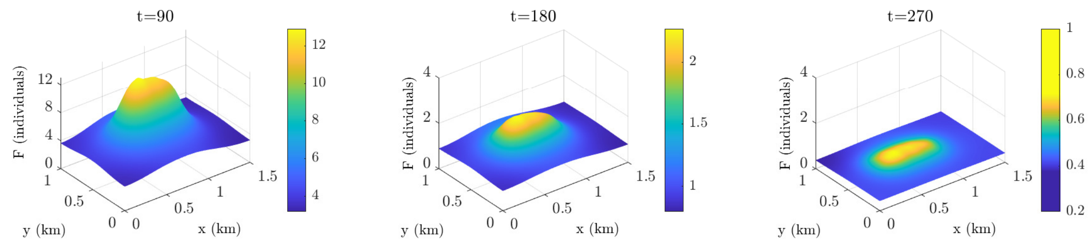
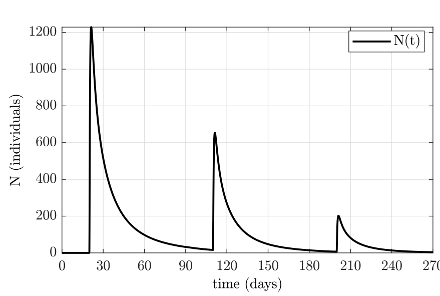


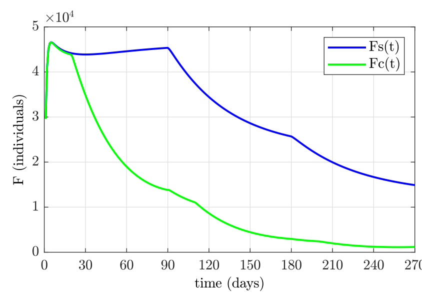
Figure 7. Strategy releasing sterile mosquitoes using method 1 (SM1). Fertilized female distribution over domain at selected times

Strategy releasing sterile mosquitoes using method 1 – (SM1): Consists of releasing sterile mosquitoes in each seasonal subperiod based on method 1, exactly on days 20, 110, and 200, when adult populations start to decline, as can be seen in **Figure 8(b)**. **Figure 7** again shows that the relative distribution is maintained over time, although the population decreases much more with this strategy, (see also Figure 8(b)). **Figure 8(a)** shows that the sterile mosquito population has almost disappeared when the next release is made, and **Figure 8(c)** shows that also in this case, most of the effect is achieved in the first subperiod.

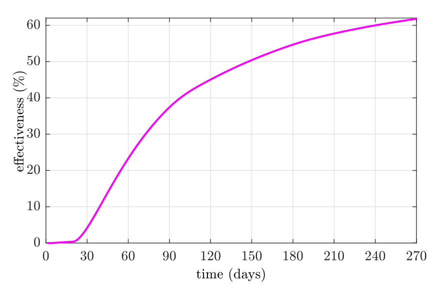
Strategy releasing sterile mosquitoes using method 2 – (SM2): In each seasonal subperiod, the sterile mosquitoes were released based on method 2, exactly during days $20, 21, \ldots, 29$; $110, 111, \ldots, 119$ and $200, 201, \ldots, 209$. There is little variability in the behavior of the populations concerning the previous strategy (SM1), showing that the effectiveness of both strategies is also very similar (see **Table 4**).



(a)



(b)



(c)

Figure 8. Strategy releasing sterile mosquitoes using method 1 (SM1). (a) Sterile mosquito population at node 1004, very close to node 1008 which is a release node. (b) Fertilized females throughout the whole domain without control ($F_s(t)$) and with control ($F_c(t)$). (c) Effectiveness of the strategy.

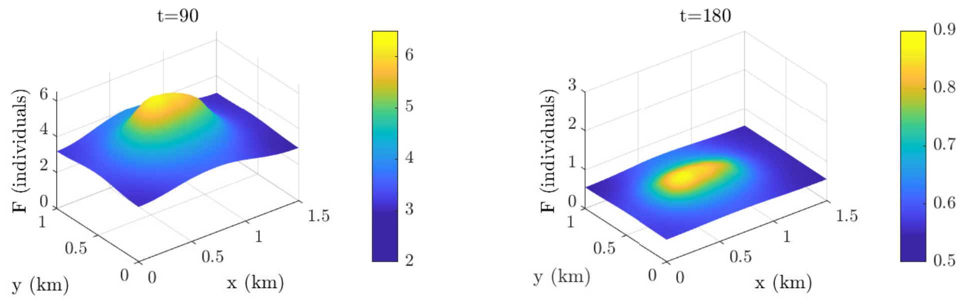
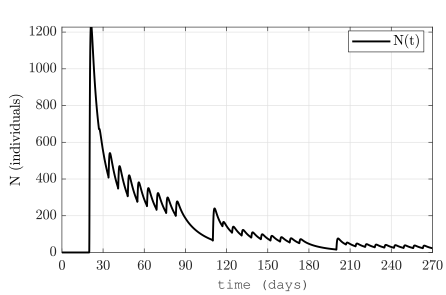


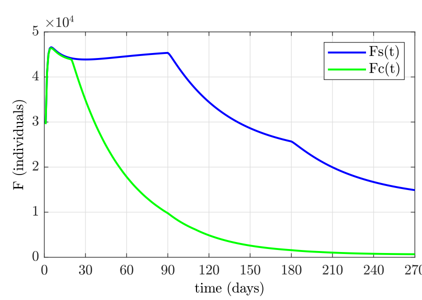
Figure 9. Strategy releasing sterile mosquitoes using method 3 (SM3). Fertilized female distribution over domain at selected times.

Strategy releasing sterile mosquitoes using method 3 – (SM3): In this strategy, the sterile mosquitoes were applied in each seasonal subperiod based on method 3, on days $20, 27, \ldots, 83$; $110, 117, \ldots, 173$ and $200, 207, \ldots, 242$. Comparing with strategies (SM1) and (SM2), **Figure 9** shows a faster decrease in the population of fertilized females as time passes, which is also observed in **Figure 10(b)**. **Figure 10(a)** shows a greater variability of the population of sterile males concerning the previous strategies, but as in strategies (SM1) and (SM2), with a tendency to disappear, and **Figure 10(c)** is similar to the effectiveness of the strategy (SM1).

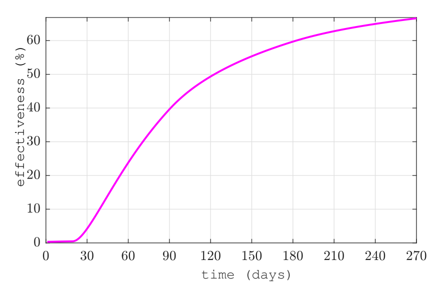
Strategy releasing sterile mosquitoes using method 4 – (SM4), 5 – (SM5) and 6 - (SM6): These strategies are similar to strategies (SM1), (SM2), and (SM3), respectively, but using the values of δ_n, determined by methods 4, 5, and 6, respectively. **Figure 11** illustrates the (SM4) strategy, and as in the previous cases, it shows that the relative distribution is maintained over



(a)



(b)



(c)

Figure 10. Strategy releasing sterile mosquitoes using method 3 (SM3). (a) Sterile mosquito population at node 1004, very close to node 1008 which is a release node (b) Fertilized females throughout the whole domain without control ($F_s(t)$) and with control ($F_c(t)$).

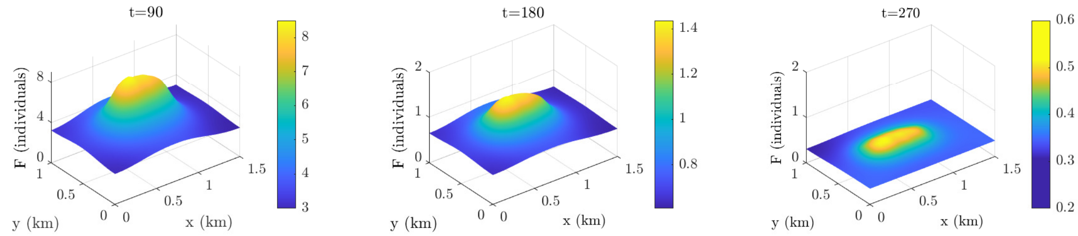


Figure 11. Strategy releasing sterile mosquitoes using method 4 (SM4). Fertilized female distribution at selected times

time, decreasing the population a little faster than the (SM1) and (SM2) strategies, but not faster than the (SM3) strategy, which can also be observed in **Figure 12(b)**. **Figure 12(a)** shows that, for strategy (SM4), although at the beginning the sterile mosquito population is bigger (more mosquitoes were released for this strategy), during the second and third subperiods this population is pretty much the same as in strategy (SM1), and **Figure 12(c)** is similar to the effectiveness of the strategy (SM1).

Table 4 summarizes the results of the strategies involving sterile mosquitoes, including the number of mosquitoes released and the effectiveness only in the first subperiod. Thus, to reduce 40.01 % of fertilized females in the first subperiod, we need 876 139 sterile mosquitoes.

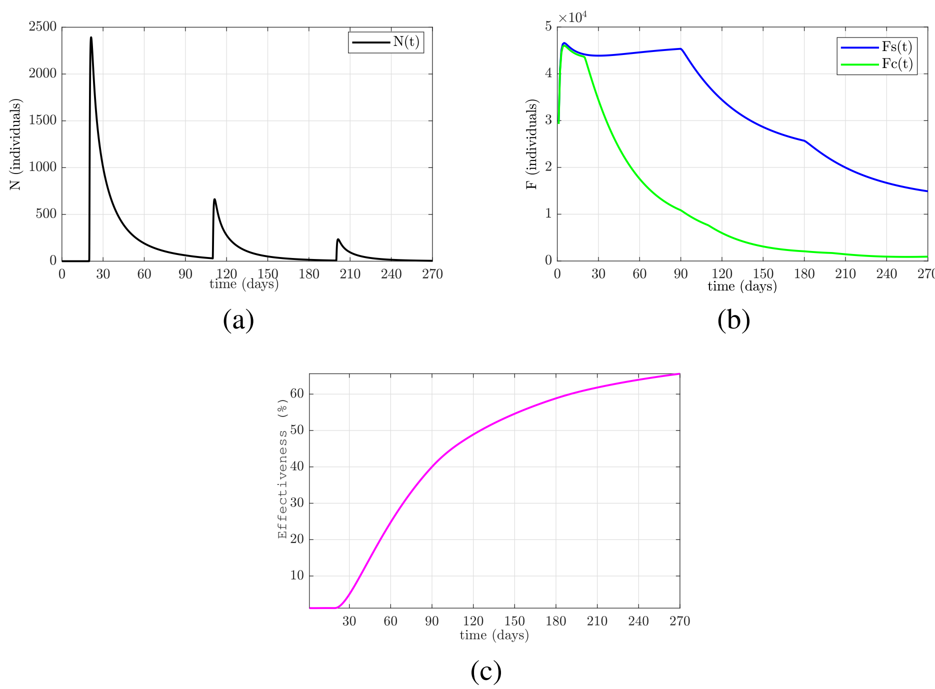


Figure 12. Strategy releasing sterile mosquitoes using method 4 (SM4). (a) Sterile mosquito population at node 1004, very close to node 1008, which is a release node. (b) Fertilized females throughout the whole domain without control ($F_s(t)$) and with control ($F_c(t)$).

4.3. Simulations combining control strategies

Now we combine chemical control and sterile mosquitoes, using only the best effective SIT strategy in the first subperiod, which is (SM4) strategy (see Table 4).

Strategy combining larvicides and sterile mosquitoes – (LSM4): This strategy combines strategy (L) and strategy (SM4).

Strategy combining insecticides and sterile mosquitoes – (ISM4): Here, we combine strategy (I) and (SM4), the days of insecticide application are 1, 5, 9, 13, 17, 91, 95, 99, 103, 107, 181, 185, 189, 193 and 197, and sterile mosquitoes on days 20, 110 and 200, to avoid killing sterile mosquitoes and decreasing effectiveness.

Strategy combining larvicides, insecticides, and sterile mosquitoes – (LISM4): Finally, strategies (L), (I), and (SM4) are combined. We must be careful when choosing the days on which each control will be applied[2]. We placed the larvicide on days 0, 90, and 180, the insecticide on the same days as the previous strategy, and the sterile mosquitoes on days 21, 111, and 201. **Figure 13** shows that the relative distribution is maintained over time, and it can be seen that the population decreases even faster than in all the previous strategies, which is also observed in **Figure 14(a)** and what is eventually perceived in the effectiveness increase (see **Figure 14(b)**). In this case, the graph corresponding to the dynamics of the release of sterile mosquitoes is similar to the graph of the strategy (SM4), and therefore it is omitted.

5. Cost-effectiveness analysis

Now, our interest is to compare the strategies described in the previous sections to determine "the best" strategy to control the mosquito population, and to do it, **Table 5** summarizes the data for each strategy. The costs of larvicides and insecticides were quoted according to the characteristics of the application in our study region and according to the number of applications[3], and the cost of the sterile mosquitoes includes the cost per mosquito and a fixed cost per release [23]. The effectiveness was calculated using the **Equation 10** for each strategy.

[2] Combining these strategies in the other order (ILSM4) gives smaller effectiveness.

[3] The cost of applying larvicides and insecticides was provided by the company *Sanicontrol*, located in the city of Cali, Colombia.

Table 4. Number of sterile mosquitoes release per each subperiod. Ht: High temperature, Mt: Medium temperature and Lt: Low temperature.

Strategy	Ht		Mt	Lt	Total
	Mosqu. (#)	Effect. (%)	Mosqu. (#)	Mosqu. (#)	Mosqu. (#)
SM1	457 806	36.49	238 048	73 481	769 336
SM2	486 124	37.89	204 361	65 597	756 085
SM3	876 139	39.62	189 398	85 510	1 151 047
SM4	876 139	40.01	238 048	85 510	1 199 697
SM5	876 139	38.65	238 048	85 510	1 199 697
SM6	876 139	37.81	238 048	85 510	1 199 697

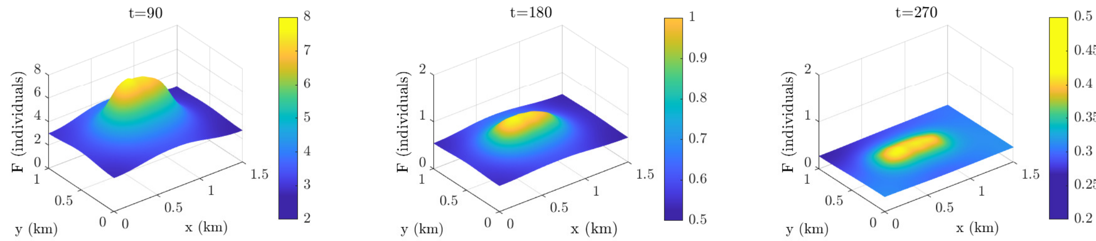
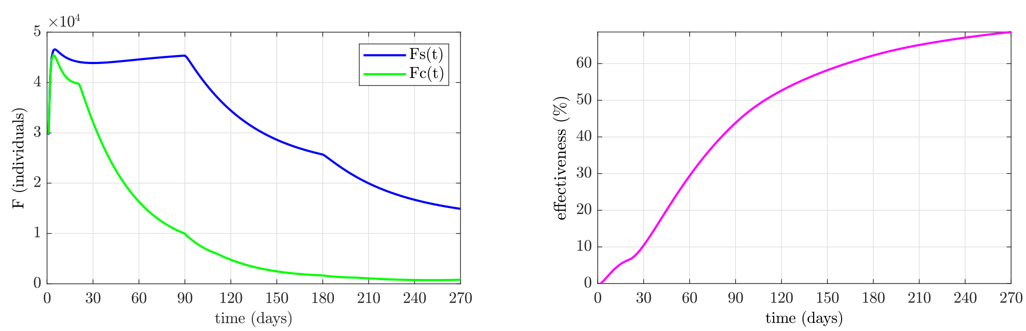


Figure 13. Strategy combining larvicides, insecticides, and sterile mosquitoes (LISM4). Fertilized female distribution at selected times.

According to the effectiveness achieved, strategy (LISM4) is "the best", but this strategy is expensive compared to the costs of the others; moreover, according to costs, strategy (I) is "the best" (the cheapest), but its effectiveness is not so good. It would be better to consider both criteria simultaneously to choose the best strategy, for which it is recommended to do an economic evaluation called cost-effectiveness. One of the indicators used to perform this analysis is the "Average Cost-Effectiveness Ratio" (ACER) given according to [29] by:

$$\text{ACER} = \frac{\text{Strategy cost}}{\text{Strategy effect}}. \tag{11}$$

We carry out the cost-effectiveness analysis considering the effectiveness and the cost of each strategy. The last column of Table 5, shows the ACER index for each strategy; from it, we can see that the best cost-effectiveness strategy (cheapest per percentage of effectiveness), is the (SM1) (sterile mosquitoes using method 1) (see Figure 7 and **Figure 8**).



(a) (b)

Figure 14. Strategy combining larvicides, insecticides, and sterile mosquitoes (LISM4). (a) Fertilized females throughout the whole domain without control ($Fs(t)$) and with control ($Fc(t)$). (b) Effectiveness of this strategy.

Table 5. Effectiveness, cost and ACER index per strategy

Strategy	Costs (US $)	Effect. (%)	ACER
L	317	8.16	38.35
I	3606	6.69	539.06
IL	3923	14.53	270.02
SM1	650	61.33	10.60
SM2	859	62.31	13.78
SM3	1180	65.61	17.71
SM4	1000	65.60	15.24
SM5	1219	64.90	18.79
SM6	1219	65.89	18.50
LSM4	1317	68.36	19.26
ISM4	4606	65.87	69.93
LISM4	4923	68.62	71.74

Another indicator used for cost-effectiveness analysis is the "Incremental Cost-Effectiveness Ratio" (ICER),

$$\text{ICER} = \frac{\text{Cost difference between strategies } x \text{ and } y}{\text{Effect difference between strategies } x \text{ and } y},$$

used to compare mutually exclusive strategies [29], here the question to answer is, what is the cost per unit of the additional benefit earned with the new strategy? To calculate this indicator, the strategies are ordered by effectiveness, then those that have a higher cost than the next are discarded, leaving the strategies to be compared using the ICER index (see **Table 6**).

Then, we discard the strategies with a higher ICER index than the following (see **Table 7**). With these results, we conclude that the best cost-effectiveness strategy is again (SM1), *(sterile mosquitoes using method 1)* and for example, comparing it to strategy (SM4) (see Figure 11 and **Figure 12**), the cost per additional unit of effectiveness is 81.94 %.

Table 6. Strategies to compare according to the ICER index

Strategy	Costs (US $)	Effect (%)	ICER
L	317	8.16	38.85
SM1	650	61.33	6.26
SM2	859	62.31	213.00
SM4	1000	65.60	42.90
SM3	1180	66.61	178.18
LSM4	1317	68.36	78.30
LISM4	4923	68.62	13 870.54

Table 7. Strategies selected by ICER index.

Strategy	Costs (US $)	Effect. (%)	ICER
SM1	650	61.33	10.60
SM4	1000	65.60	81.94
LSM4	1317	68.36	114.85
LISM4	4923	68.62	13 870.54

6. Discussion

The *Aedes aegypti* mosquito is geographically established in many parts of the world and is considered the main vector for the transmission of diseases such as zika, dengue, and chikungunya. This mosquito mainly moves over urban areas of cities with climates suitable for its proliferation, but it has been shown that they can adapt even at lower temperatures, which implies that outbreaks cannot be ruled out in any seasonal period in tropical and subtropical areas.

In particular, Cali is a city mainly with medium and high temperatures and has presented several dengue epidemics in recent years, proving that the control mechanisms used by public health agencies have not been sufficient to stop them. The same has happened in other countries around the Americas, such as Brazil and Paraguay [30]. Larvicides and insecticides are useful and effective, but their use is not suitable for long periods due to the damaging effects they cause on the environment and human health, in addition to the resistance that mosquitoes can acquire towards these products. The SIT can be an alternative to control methods as an ecological and sustainable tool. For a successful application of this technique, it is necessary to have not only a broad knowledge of the mosquito, but also a knowledge of the physical and environmental characteristics of the place where it is to be implemented, and for this, it is very useful to have temporal-space models that allow simulating these such complex dynamics in a short time and at a low cost.

The model that we present is the result of modifications and improvements of models that only analyze the dynamics in time, as studied in [2]; by introducing the spatial components, we obtain a reaction-diffusion system to model the dispersion of the *Aedes aegypti* mosquito under the influence of different control methods that are applied to a specific region, a rectangle with marked geographical delimitation, not so extensive and with a heterogeneous landscape suitable for our study.

For the simulations, we used adequate numerical methods for this type of problem, emphasizing that due to the complexity of the system, the execution times of each scenario were quite long (on average 20 hours per simulation on a computer with a processor Intel Core i7-8th Gen, 2.60 GHz, 4.30 GHz Turbo and with 32 MB of RAM), which could be seen as a disadvantage of the model; but it can be improved by having more suitable and better potentialized equipment that offers a considerable reduction of this time.

Numerical simulations show us the importance of considering certain environmental factors such as the temperature, which significantly affects the efficiency of the strategies implemented, mainly those involving SIT on a space-time scale. For each strategy, we simulate the evolution of the mosquito populations with quarterly variations in temperature using appropriate data; we note that, without applying controls, mosquito populations tend to have different values (limits) in each subperiod, which decrease with temperature. This is very surely related to the change in the set of entomological parameters, due to the decrease in temperature of each subperiod, which also produces a decrease in the survival threshold of mosquitoes given by the temporal model used as the basis for our model. Indeed, using equation (4) of [2], the values of this threshold for the subperiods of the high, medium, and low temperatures are 89.0, 54.12, and 38.42, respectively. All this could explain the lower incidence of dengue in cities with lower average temperatures [31]. Due to these factors, the effectiveness of the strategies used was better (more than $2/3$ of the total effectiveness) during the high-temperature subperiod.

From Table 5, we can see that the most effective and, in turn, the most expensive strategy is the strategy (LISM4), which combines all the controls. On the other hand, the cheapest strategy but with low effectiveness is the strategy (L), perhaps since larvicide is only applied in breeding sites; and the least effective strategy is the strategy (I) and it is also expensive. It is important to note that, although larvicide has a significant and immediate effect on the decrease of the aquatic population, it is not long-lasting and its effect on the decrease of the fertilized female population is very small. Perhaps an increase in the frequency of larvicide application will significantly increase its effect on the population of fertilized females.

We also observe that the effectiveness that traditional controls (strategy (I) or (L)) add to the strategies that include sterile mosquitoes (strategies (LSM4) and (ISM4)) is very low, but they do increase the cost, mainly the insecticide. All of these show the need of applying other alternatives, such as sterile mosquitoes, without combining them with chemical controls.

It is worth noting that the effectiveness does not vary considerably in strategies that involve sterile mosquitoes, (using either a single or a periodic release during each seasonal period). However, in the results of the cost-effectiveness analysis carried out with the indicators ACER and ICER, we show the goodness of the strategies in which all the mosquitoes are released in a single release; in both cases, the best strategy is (SM1) (see Table 5), although this is a strategy that in practice is limited by the fact that the number of wild males present at the time must be known in a very approximate way, leading research on ways to make these estimates. In practical settings, reliable estimates of the current size of the wild mosquito population can be obtained by employing the so-called Mark-Release-Recapture technique [32].

7. Conclusions

The importance of considering temperature both in the population dynamics of the *Aedes aegypti* mosquito and in the effectiveness of the controls used to reduce its growth is evident. We should also mention that the inclusion of the space variable in the dynamic analysis of the population growth of the *Aedes aegypti* mosquito and in the action of the controls allows simulating the effect of the controls in specific sites of the mosquito's habitat, such as breeding sites and points of greatest human-mosquito interaction. Finally, our study also shows that the sterile mosquito release technique (SIT) may be a more cost-effective way to control the *Aedes aegypti* mosquito than traditional chemical techniques and provides baseline information for the development of more rational approaches to vector control.

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Conflict of interest

The authors declare that they have no conflict of interest.

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Estrategias de control en la dinámica poblacional espacial del vector *Aedes aegypti* usando mosquitos estériles e insecticidas

Resumen: En este trabajo analizamos en un área determinada, la dinámica poblacional del mosquito *Aedes aegypti*, transmisor de diversos virus como el dengue, zika y chikungunya, a partir del uso de diversas técnicas de control. Para ello, utilizamos un modelo de reacción-difusión que considera varias características ambientales como la temperatura y la forma del paisaje. Inicialmente, hacemos este análisis utilizando larvicidas, insecticidas y la técnica SIT (liberación de insectos estériles) por separado.

Simulamos diferentes escenarios de control con el uso de métodos numéricos apropiados, probamos combinaciones de estas técnicas y determinamos la eficiencia de cada estrategia de acuerdo con la reducción general en el número de hembras fertilizadas debido a la técnica aplicada. Posteriormente, mediante un análisis de costo-efectividad, verificamos que la liberación de mosquitos estériles al inicio de cada período estacional es la mejor estrategia para controlar la población de *Aedes aegypti*.

Palabras Clave: *Aedes aegypti*, Análisis de costo-efectividad, Insecticida, Larvicida, Modelo de reacción-difusión, Mosquitos estériles, .

Estratégias de controle na dinâmica espacial da população do vetor *Aedes aegypti* usando mosquitos estéreis e inseticidas

Resumo: Neste trabalho, analisamos a dinâmica populacional do mosquito *Aedes aegypti*, transmissor de vários vírus como dengue, zika e chikungunya, em uma determinada área, com base no uso de diversas técnicas de controle. Para isso, usamos um modelo de reação-difusão que considera várias características ambientais, como temperatura e forma da paisagem. Inicialmente, fazemos essa análise usando larvicidas, inseticidas e a técnica SIT (sterile insect release) separadamente.

Simulamos diferentes cenários de controle usando métodos numéricos apropriados, testamos combinações dessas técnicas e determinamos a eficiência de cada estratégia de acordo com a redução geral no número de fêmeas fertilizadas devido à técnica aplicada. Posteriormente, através de uma análise de custo-efetividade, verificamos que a liberação de mosquitos estéreis no início de cada período sazonal é a melhor estratégia para controlar a população de *Aedes aegypti*.

Palavras-chave: análise de custo-efetividade, *Aedes aegypti*, Inseticida, Larvicida, Modelo de reação-difusão, Mosquitos estéreis.

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