Pontificia Universidad
JAVERIANA
Bogotá

Univ. Sci. 27(3): 315–331, 2022
doi: 10.11144/Javeriana.SC273.eeof

ORIGINAL ARTICLE

Extrapolation errors of force transducer curve fitting equations

Riham Hegazy*[1], Gouda Mohammad[1], Magdy Ibrahim Mohamed[1]

Edited by
Juan Carlos Salcedo-Reyes
salcedo.juan@javeriana.edu.co

1. National Institute of Standards, NIS, Egypt

*rihamhegazy@yahoo.com

Received: 03-05-2022
Accepted: 30-09-2022
Published online: 12-12-2022

Citation: Hegazy R, Mohammad G, Mohamed MI. Extrapolation errors of force transducer curve fitting equations, *Universitas Scientiarum*, 27(3): 315–329, 2022.
doi: 10.11144/Javeriana.SC273.eeof

Funding: n.a.

Electronic supplementary material: n.a.

Abstract

Calibration laboratories often face the challenge of the impossibility to perform full capacity range calibration of their force transducers, particularly below 10 % of the force transducer's capacity. Sometimes these laboratories use curve fitting extrapolation to estimate and predict force transducer behavior within uncalibrated capacity ranges. This work deals with the study of extrapolation errors in force transducers to know and estimate prediction accuracies when using extrapolation for force transducer calibration in ranges below 10 % and between 50 % and 100 % of the transducer's capacity. The results of this study showed that the magnitude of the extrapolation error is very close to the magnitude of the reproducibility error within calibrated capacity ranges in the laboratory.

Keywords: force transducer; calibration range; extrapolation error; accuracy; uncertainty

1. Introduction

In force measurement, it is central to study the factors likely affecting force transducer calibration results. These factors include: the load cell calibration method [1], the creep and creep recovery time [2], and the force transducer's low range behavior [3, 4]. Force metrology laboratories often have a limited capacity of primary standards, and sometimes these laboratories are requested to conduct measurements in a range twice (or even several times) as high as their actual calibration capacity.

This problem arises from the steady progress in industry, technology, and structural engineering coupled with the consequent demand for very high load measurements and the need for parameters to ensure the accuracy and precision of these measurements. ISO 376 mentioned the minimum force for the calibration of the proving instrument considering the indicator's resolution. Namely, the minimum force applied to a force-proving device shall comply with the two following conditions:

(i) The minimum force shall be greater than or equal to:

- $4000 \times r$ for class 00,
- $1000 \times r$ for class 1, and
- $500 \times r$ for class 2.

(ii) The minimum force shall be greater than or equal to 0.02 Ff.

Where r is the resolution of the transducer and Ff is the max capacity of the transducer.

Moreover, a laboratory may also be required to conduct measurements in the smallest range mentioned in the ISO standard but may lack the ability to calibrate its force transducers within this range. Laboratories may choose to provide a broad force standard capacity, ranging from minuscule through medium and up to very high force values. This solution is ideal but requires considerable economic investment. Another solution is to calibrate the transducer within a fraction of the force range and to obtain a relationship or equation correlating the assessed forces and their corresponding outputs. This relationship can be generalized to the full range of the transducer. Before applying this method, laboratories must study the factors governing the difference between the behavior of the transducer experienced by the experiment and the calculated behavior through the correlation equation. The consequent increase in the uncertainty associated with the results also requires evaluation. Recently, some studies have addressed this issue [5, 6, 7].

This study deals with a partial range calibration for three force transducers. Extrapolation was done to determine the output of these transducers within the required remaining range. Furthermore, the differences between the experimental and the calculated results were assessed and discussed, considering the accompanying result uncertainties.

2. Experiment design

Three force proving instruments with capacities of 200 N, 500 N, and 1000 N were calibrated. Each force proving instrument was calibrated in its full range with two additional partial range calibrations. The first partial calibration range went from 1 % to 10 % of its maximum force, and the second range spanned 5 % to 50 % of the transducer's maximum force. Thus, each instrument was calibrated for three ranges through the application of force and records the output corresponding to the instrument at each force, as follows:

- Partial range from 1 % to 10 % of maximum force with a 1 % step.
- Partial range from 5 % to 50 % of maximum force with a 5 % step.
- Full range from 10 % to 100 % of maximum force with a 10 % step.

Measurements for each range were taken according to ISO 376 [8] at (20 ± 1) °C, using a readout device DMP 40 s2. The load is applied through a 1000 N dead weight machine (DWM) and small standard weights with uncertainty of ± 85 ppm as shown in **Figure 1** and **Figure 2**.

Employing the full range calibration results and the obtained calibration curve (third order polynomial), A set of force proving instrument outputs were calculated for the partial range from 1 % to 10 % of the maximum force. By comparing the calculated values with the actual measurements within this partial range, the error of the derived values and their deviation from the measurements in the laboratory were calculated. These deviation values were compared with the maximum allowable relative error values in ISO 376, for each class of the force proving instruments.

The calibration curves (third order polynomials) from the readings within the 5 % to 50 % of the maximum force range of each device served to predict force output values within the subsequent 50 % to 100 % of the maximum force range of each instrument. The errors of the calculated values and their deviation from the measurements in the laboratory were estimated. The diffraction values were compared with the maximum allowable relative error values represented by the uncertainty

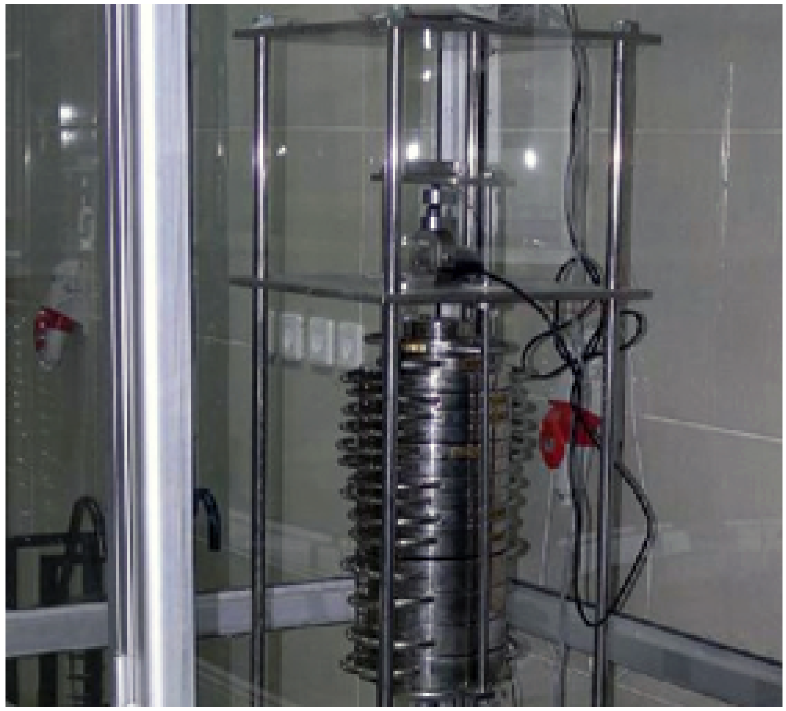

Figure 1. 1000 N DWM.

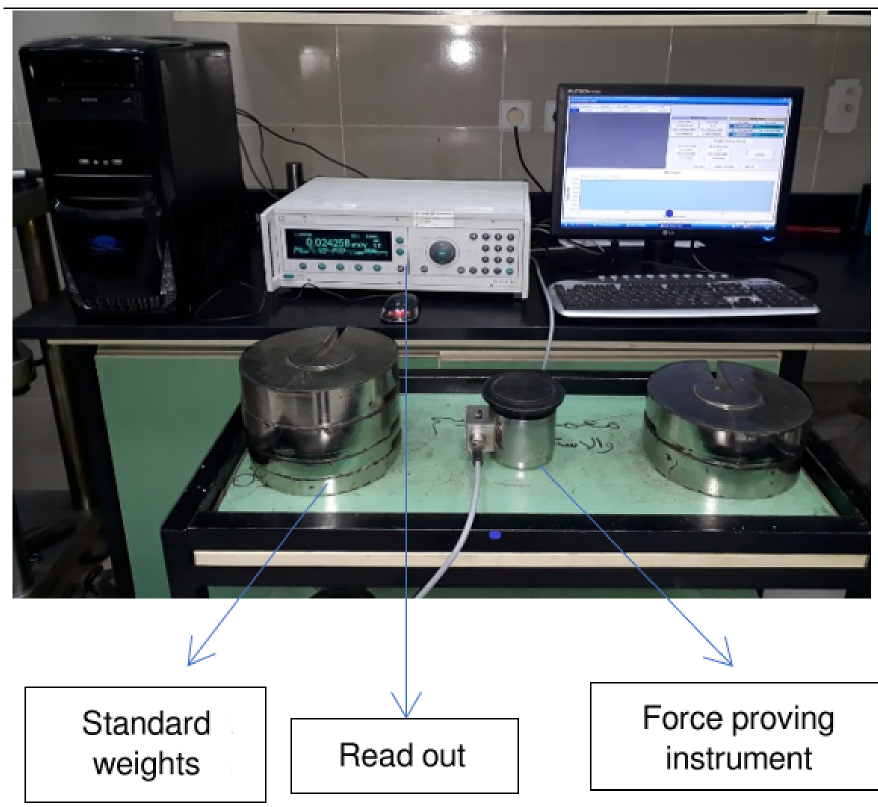

Figure 2. Standard weights setup.

elements mentioned in ISO 376 to: (i) determine the extent to which the half-range calibration can be used, (ii) evaluate the suitability of the equation from the whole range calibration curve, and (iii) ponder the validity of the results in laboratories unable calibrate the high force measurements.

3. Theoretical background

For any general fitted relation between the independent variable X and its dependent variable Y, we can assume the difference between a given value Y_i and its corresponding calculated value X_i, determined from the fitted curve C, as D_i. Consequently, the value set $X_1, X_2, \ldots, X_n$ will have deviations $D_1, D_2, \ldots, D_n$ [9]. A measure of the curve C accuracy of fit to the given data is provided by the quantity ${D_1}^2 + {D_2}^2 + \cdots + {D_n}^2$. If this quantity is small the fit is accurate. So, the best fitting curve, requires that:

$$\sum_{n=1}^{n} {D_i}^2 \text{ is minimum.}$$

The least square method was used to obtain the calibration equations via third-degree polynomials which satisfy the minimum sums of squares of the residuals.

All the uncertainty values were estimated according to ISO 376:2011(E).

The relative deviations $(R.D)$ of the calculated load cell outputs from to the actual outputs of the force proving instrument within the studied partial ranges were calculated using equation 1:

$$R.D = \frac{O_{\text{ext}} - O_{\text{A}}}{O_{\text{A}}} \times 100 \tag{1}$$

Where O_{ext} is the extrapolated value of the force transducer's output, and O_{A} is the actual output value of the force transducer obtained by calculating the average responses of the transducer corresponding to each applied force through the experiment.

4. Results and discussion

4.1. Linearity of the force proving instruments

The non-linearities of the force proving instruments were calculated from **Equation 2**

$$\text{Non-linearity} = \frac{X}{X_N} - \frac{F}{F_N} \tag{2}$$

Where X is the deflection with increasing test force F, X_N deflection corresponding to the maximum calibration force, and F_N is the maximum calibration force.

The non-linearity of the three force proving instruments was highly variable across tests. **Figure 3**, **Figure 4**, **Figure 5**, **Figure 6**, **Figure 7**, and **Figure 8** show the results of the non-linearity of the three force proving instruments

The 200 N force proving instrument revealed the most non-linear readings below 10 % of its maximum capacity (Figure 3); however, it did not exceed the non-linearity value of −0.001, and when extrapolating non-linearity using a third-order polynomial equation, deviations diverted from the actual mean deflection by 0.000 73.

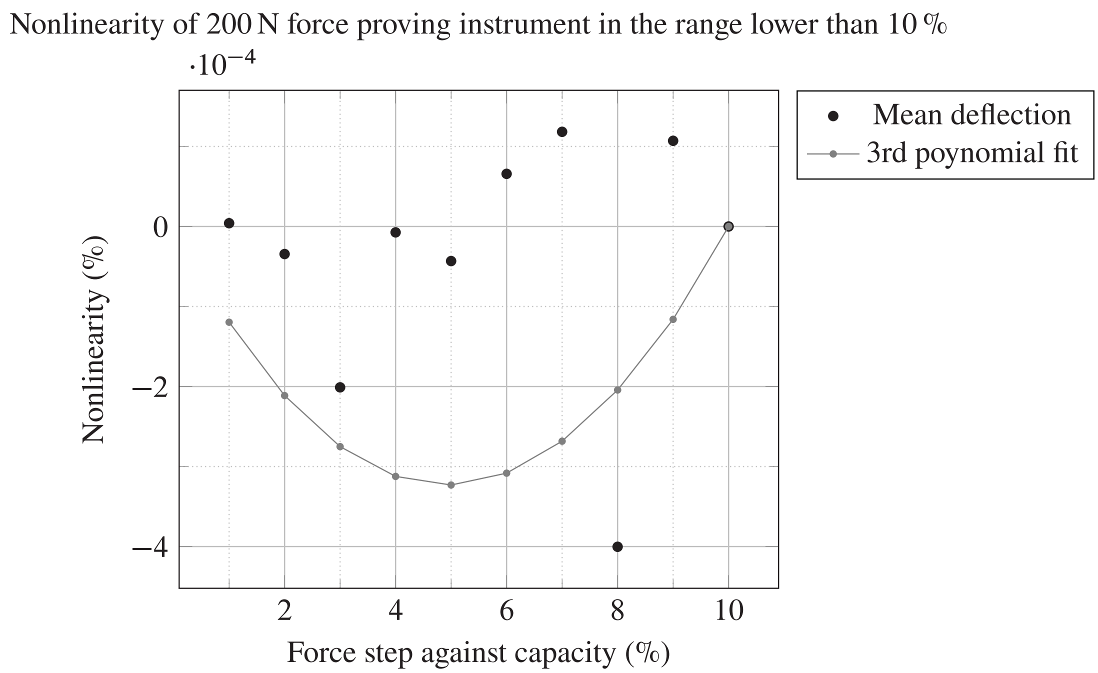

Figure 3. Non-linearity of the 200 N force proving instrument below 10 % of the device's capacity.

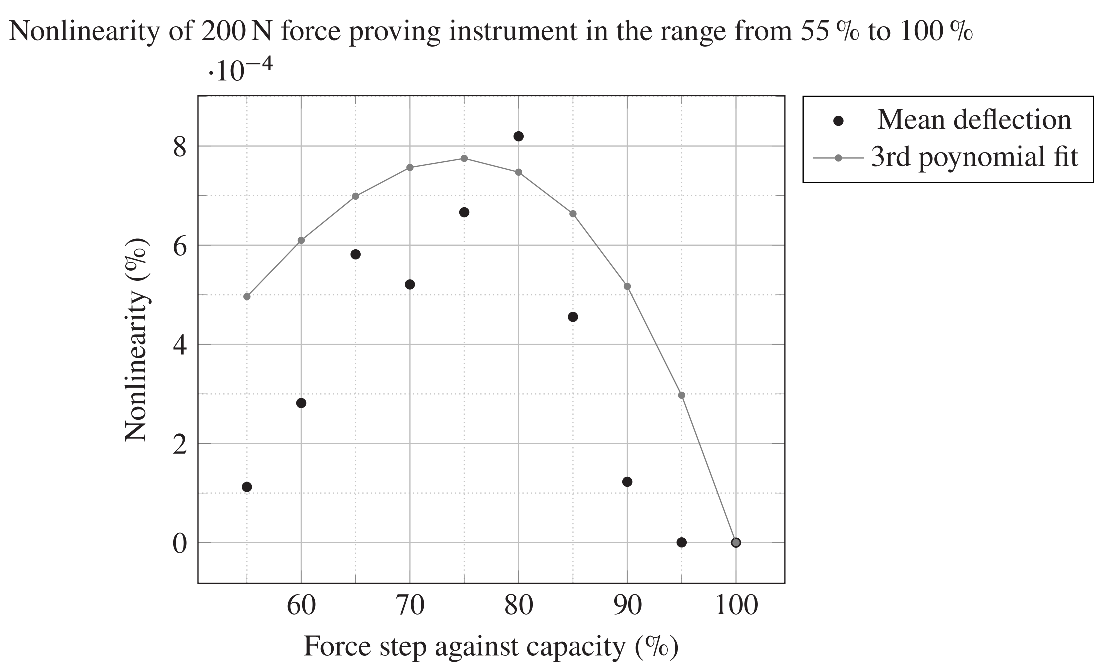

Figure 4. Non-linearity of the 200 N force proving instrument in the range from 55 % to 100 % of its capacity.

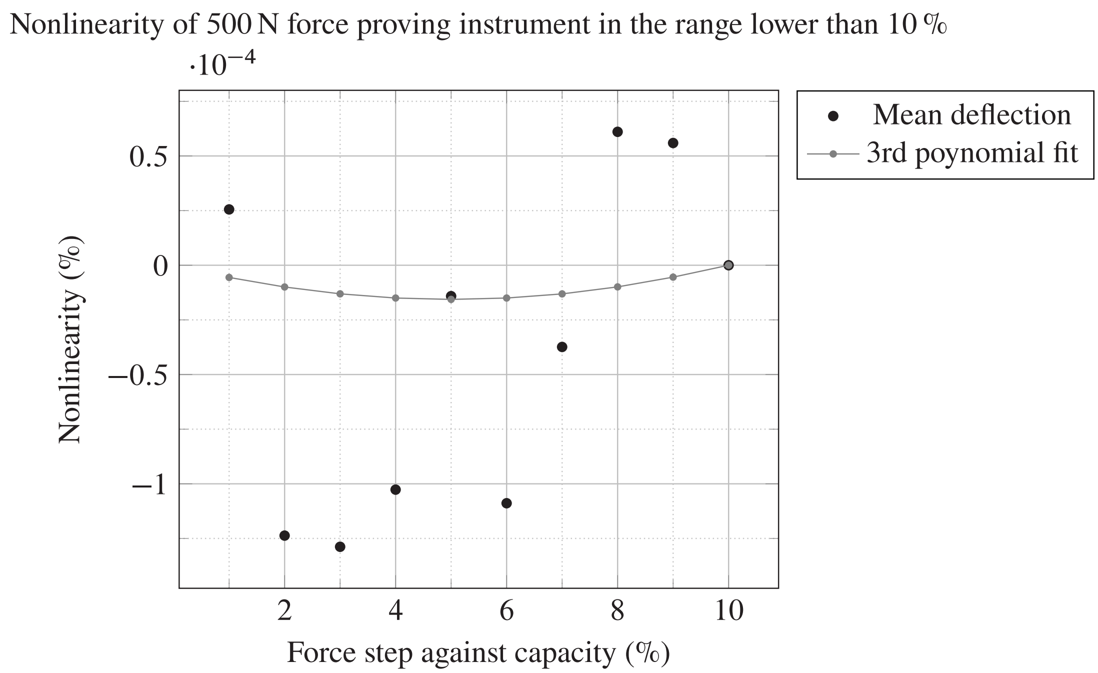

Figure 5. Non-linearity of the 500 N force proving instrument below 10 % of its capacity.

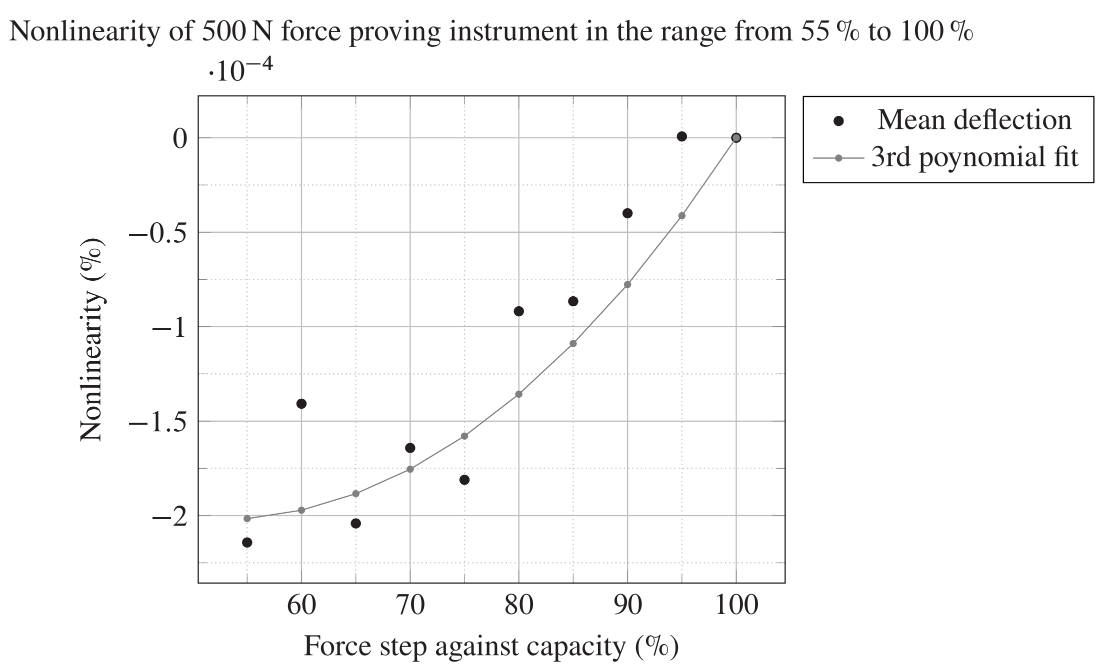

Figure 6. Non-linearity of the 500 N force proving instrument in the range from 55 % to 100 % of its capacity.

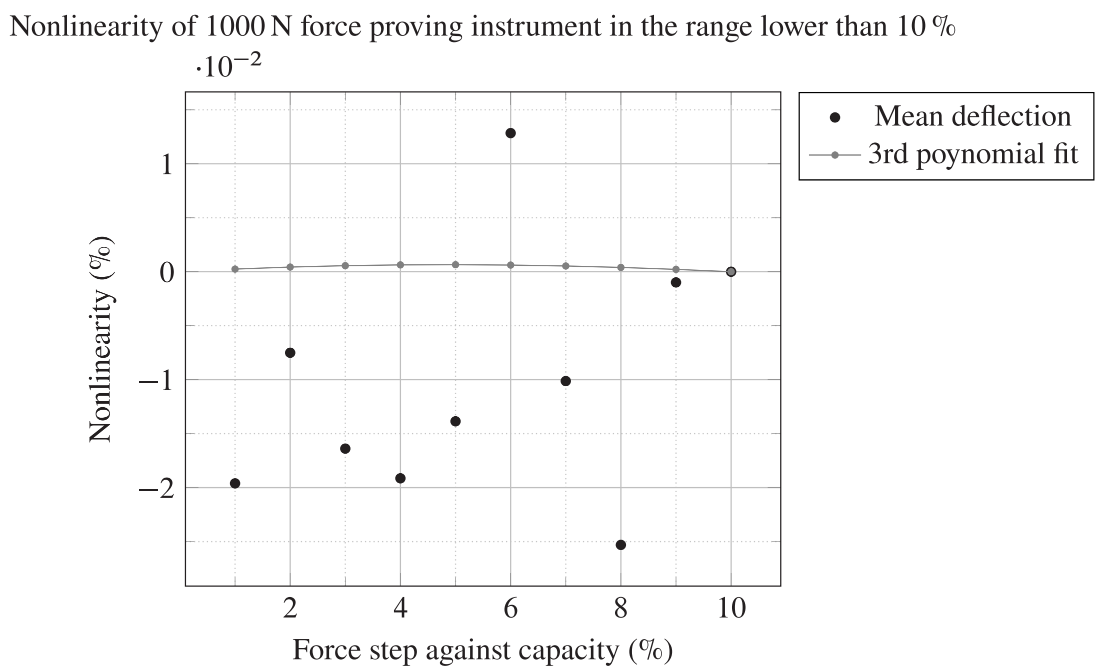

Figure 7. Non-linearity of the 1000 N force proving instrument below 10 % of its capacity.

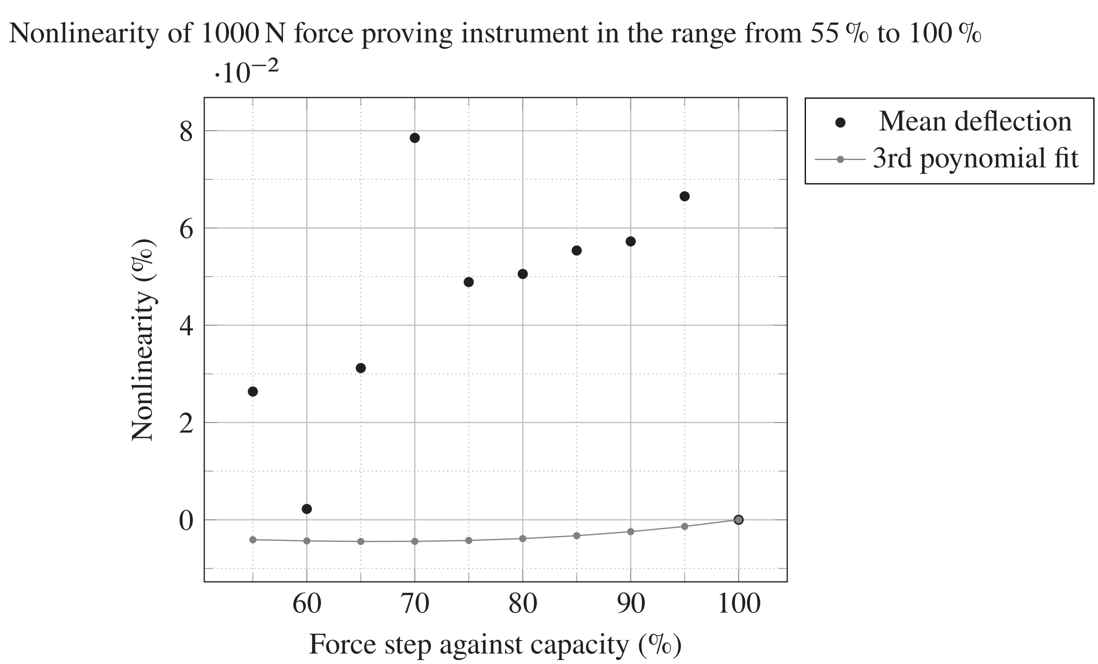

Figure 8. Non-linearity of 1000 N Force proving instrument in the range from 55 % to 100 % of its capacity.

Table 1. Relative deviations of the extrapolated values for two device capacity subranges from the actual experimental values.

Force Level(%)	Relative deviation (%)			Force Level(%)	Relative deviation (%) For load cells with capacities		
	200 N	500 N	1000 N	(%)	200 N	500 N	1000 N
1	0.0116	0.0001	0.153	55	−0.0125	0.0092	−0.0031
2	−0.0197	0.0391	−0.0051	60	0.0159	0.0001	0.0214
3	0.0234	0.0212	0.0132	65	0.0273	0.0141	−0.0125
4	−0.043	0.0043	0.0081	70	0.0231	0.0121	−0.1002
5	−0.0285	−0.0172	−0.011	75	0.0229	0.0204	−0.0473
6	−0.0533	−0.0014	−0.0591	80	0.0167	0.0152	−0.0522
7	−0.0545	−0.0132	−0.0222	85	0.0122	0.0214	−0.0661
8	0.0133	−0.0251	−0.0041	90	0.0276	0.0232	−0.0772
9	−0.0466	−0.0234	−0.0342	95	0.0339	0.0273	0.0752
10	−0.0201	−0.016	−0.0341	100	0.0421	0.0361	0.018

The non-linearity with the 500 N force proving instrument did not exceed the value of 0.000 18 in measurements of 55 % to 100 % of its maximum capacity (Figure 6), and the deviation between the third-order polynomial and the actual mean was about 0.000 12. For the 1000 N force proving instrument, maximum non-linearity did not exceed the value of 0.0004, and when extrapolating it using a third-order polynomial fitting curve, it deviated from the actual mean deflection by 0.000 46. Altogether these results indicate that the three force-proving instruments exhibit a sufficient linearity behavior and extrapolate well to the fitting third-order polynomials.

The relative deviations, for two capacity subranges, between the values obtained from the three force-proving instrument calibration equations and the actual measurements in their full capacity range are shown in **Table 1**.

Figure 9 and **Figure 10** plot the relative deviations between calculated and measured force values within two capacity sub-ranges, (*i.e.*, below 10 % and from 55 % to 100 % of the instrument capacity) respectively. For these two partial ranges, the deviations of these values were derived from the actual experimental values using equation 1.

Table 2, **Table 3**, and **Table 4** present the calibration results and classifications of the three force proving instruments according to the errors of reproducibility, repeatability, interpolation, zero, and creep.

Table 2. Calibration results and classification of 200 N force proving instrument

Force level (%)	Relative Reproducibility error	Classification class	Relative Repeatability error	Classification class	Relative interpolation error	Classification class
1	0.307 43	2	0.005 11	0	0.007 135	0
2	0.224 34	2	0.001 79	0	0.016 998	0
3	0.171 14	1	0.001 71	0	0.055 162	1
4	0.124 03	1	0.003 71	0	0.009 948	0
5	0.100 30	1	0.001 94	0	0.010 726	0
6	0.086 96	0.5	0.001 96	0	0.023 617	0
7	0.065 08	0.5	0.000 00	0	0.024 575	0
8	0.064 61	0.5	0.000 96	0	0.037 317	0.5
9	0.048 27	0	0.005 55	0	0.027 026	0.5
10	0.209 05	2	0.001 58	0	0.101 819	2
15	0.043 06	0	0.007 12	0	0.188 244	2
20	0.035 57	0	0.004 47	0	0.130 576	2
25	0.037 61	**0**	0.002 04	**0**	0.078 517	**1**
30	0.035 59	0	0.002 04	0	0.031 139	0.5
35	0.034 98	0	0.005 81	0	0.008 643	0
40	0.032 14	0	0.002 58	0	0.043 660	0.5
45	0.030 47	0	0.004 39	0	0.068 774	1
50	0.041 79	0	0.004 56	0	0.039 340	0.5
55	0.039 57	0	0.001 85	0	0.060 096	1
60	0.002 22	0	0.000 57	0	0.052 287	1
65	0.003 77	0	0.007 12	0	0.017 954	0
70	0.179 22	1	0.002 77	0	0.019 822	0
75	0.012 24	0	0.000 56	0	0.075 639	1
80	0.010 51	0	0.002 67	0	0.013 795	0
85	0.036 06	0	0.002 85	0	0.053 836	1
90	0.035 12	0	0.002 67	0	0.034 962	0.5
95	0.031 64	0	0.002 69	0	0.024 884	0
Creep error = 0.003 06 % with class 0, Zero error = 0.024 04 % with class 0						

Table 3. Calibration results and classification of 500 N force proving instrument

Force level (%)	Relative Reproducibility error	Classification class	Relative Repeatability error	Classification class	Relative interpolation error	Classification class
1	0.091 84	0.5	0.127 93	2	0.003 974	0
2	0.030 61	0	0.006 12	0	0.042 516	0.5
3	0.006 80	0	0.170 09	2	0.024 255	0
4	0.083 79	0.5	0.125 68	2	0.007 522	0
5	0.056 92	0.5	0.132 92	2	0.014 563	0
6	0.061 02	0.5	0.136 40	2	0.001 311	0
7	0.059 59	0.5	0.075 38	1	0.010 813	0
8	0.039 37	0	0.076 21	1	0.023 087	0
9	0.031 62	0	0.067 21	1	0.021 053	0
10	0.084 79	0.5	0.014 81	0	0.014 267	0
15	0.037 74	0	0.029 60	0.5	0.008 130	0
20	0.027 54	0	0.003 14	0	0.002 075	0
25	0.054 60	0.5	0.007 57	0	0.006 643	0
30	0.022 65	0	0.001 70	0	0.002 800	0
35	0.038 39	0	0.014 70	0	0.005 554	0
40	0.023 96	0	0.018 36	0	0.002 756	0
45	0.028 10	0	0.007 02	0	0.002 070	0
50	0.017 65	0	0.010 09	0	0.003 921	0
55	0.020 58	0	0.008 34	0	0.004 642	0
60	0.010 96	0	0.009 94	0	0.007 057	0
65	0.023 77	0	0.006 58	0	0.004 768	0
70	0.008 39	0	0.000 07	0	0.000 735	0
75	0.005 64	0	0.000 56	0	0.005 435	0
80	0.008 83	0	0.001 96	0	0.003 141	0
85	0.006 28	0	0.009 02	0	0.000 285	0
90	0.001 52	0	0.010 14	0	0.001 848	0
95	0.003 98	0	0.011 81	0	0.002 072	0
100	0.010 80	0	0.000 53	0	0.002 345	0

Creep error = 0.003 06 % with class 0, Zero error = 0.014 16 % with class 0

Table 4. Calibration results and classification of 1000 N force proving instrument

Force level (%)	Relative Reproducibility error	Classification class	Relative Repeatability error	Classification class	Relative interpolation error	Classification class
1	0.270 61	2	0.020 39	0	0.188 130	2
2	0.045 69	0	0.022 84	0	0.029 499	0.5
3	0.067 79	0.5	0.005 08	0	0.046 263	0.5
4	0.067 30	0.5	0.002 54	0	0.039 181	0.5
5	0.129 70	1	0.002 04	0	0.019 049	0
6	0.270 42	2	0.000 85	0	0.030 319	0.5
7	0.192 95	1	0.000 73	0	0.005 243	0
8	0.139 55	1	0.000 00	0	0.022 112	0
9	0.226 33	2	0.002 83	0	0.008 619	0
10	0.224 69	2	0.012 78	0	0.009 956	0
15	0.174 31	1	0.000 34	0	0.005 733	0
20	0.140 85	1	0.000 25	0	0.009 210	0
25	0.148 58	1	0.007 14	0	0.022 501	0
30	0.133 23	1	0.000 34	0	0.008 374	0
35	0.108 84	1	0.000 15	0	0.003 433	0
40	0.099 25	0.5	0.000 25	0	0.001 188	0
45	0.084 51	0.5	0.000 23	0	0.001 632	0
50	0.058 62	0.5	0.000 92	0	0.004 866	0
55	0.057 38	0.5	0.002 32	0	0.009 730	0
60	0.135 70	1	0.002 98	0	0.040 826	0.5
65	0.023 53	0	0.000 08	0	0.015 282	0
70	0.228 73	2	0.000 07	0	0.034 177	0.5
75	0.036 83	0	0.000 20	0	0.000 767	0
80	0.014 46	0	0.000 13	0	0.004 601	0
85	0.017 81	0	0.000 12	0	0.003 101	0
90	0.024 97	0	0.000 11	0	0.004 675	0
95	0.023 66	0	0.000 11	0	0.000 328	0
Creep error = 0.001 19 % with class 0, Zero error = 0.0091 % with class 0						

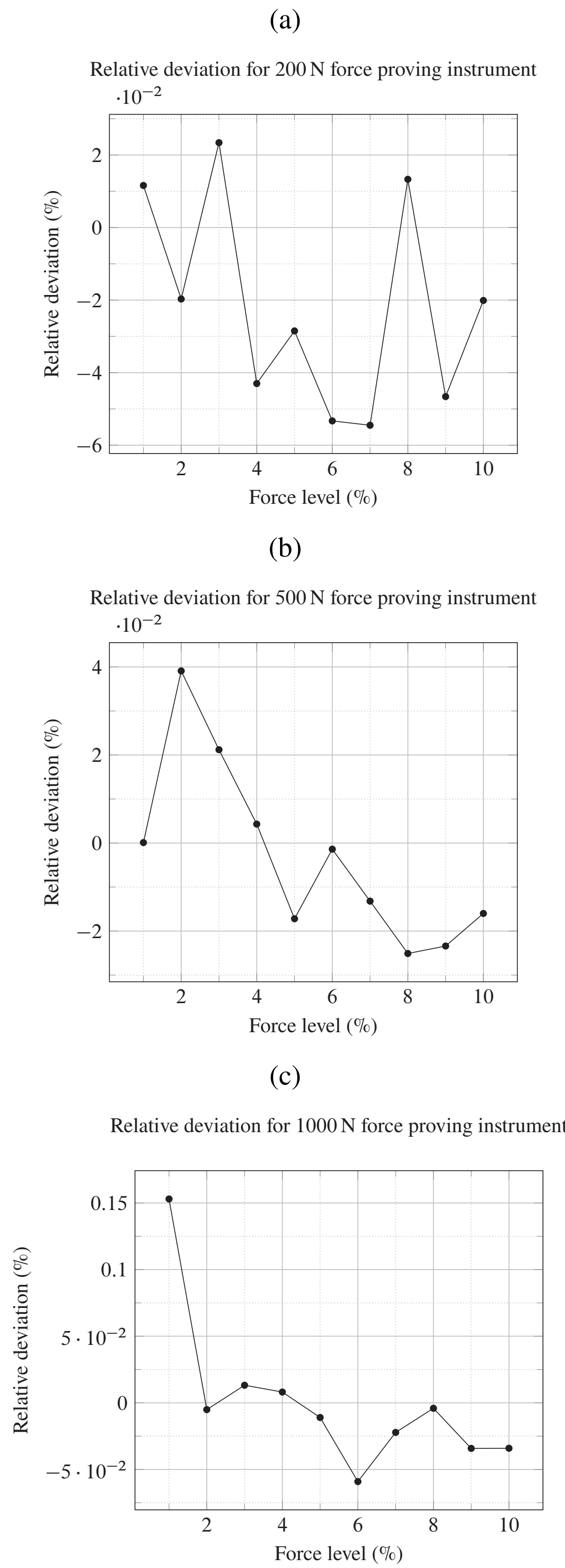

Figure 9. Extrapolation of relative deviations for strength predictions in three devices within in a range from 1 % to 10 % of their maximum capacity.

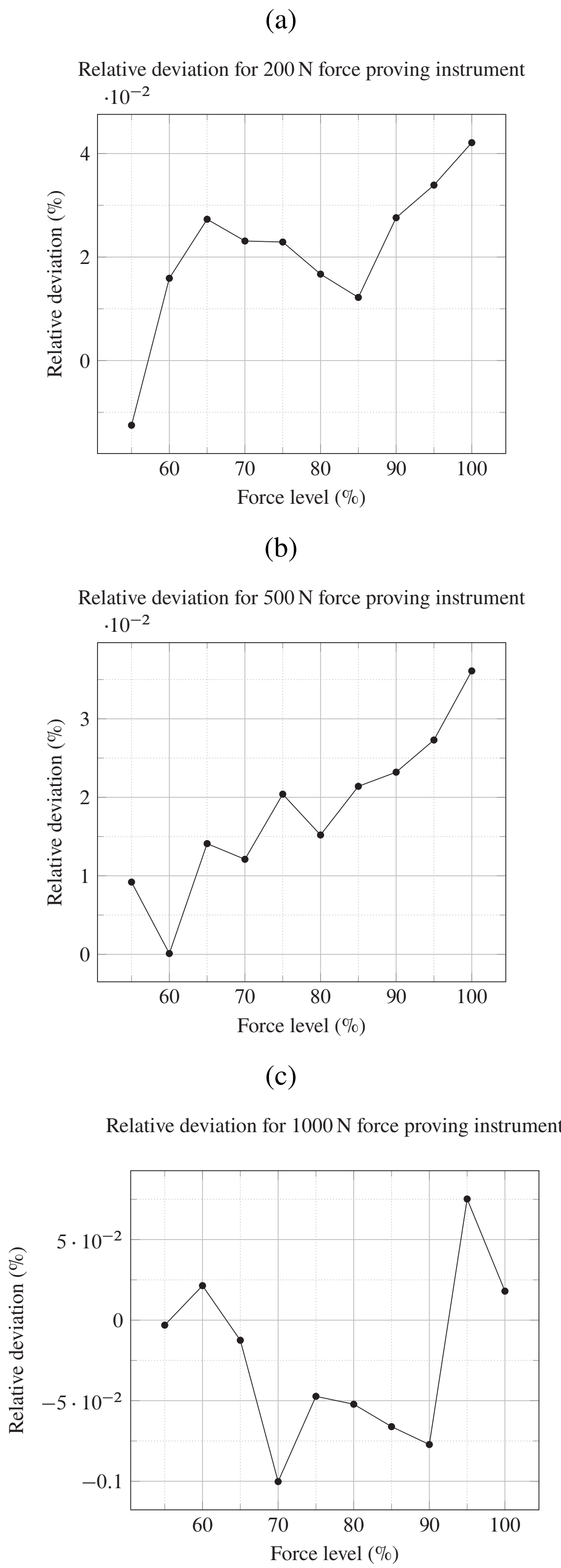

Figure 10. Extrapolation of relative deviations for strength predictions in three devices within in a range from 55 % to 100 % of their maximum capacity.

The relative deviations of the calculated values using the obtained force-proving equations were within the instrument calibration standards supported in ISO 376 [8], and None of the three force-proving instrument classes changed. For the 200 N force proving instrument, the class due to interpolation error is 2, and the maximum deviation is 0.054 %. The maximum deviation of the 500 N force-proving instrument is 0.039 %, and the class due to interpolation error is 0.5. Finally, the 1000 N force-proving instrument's class due to interpolation error is 2 with a maximum deviation of 0.153 %.

When calibrating a force-proving instrument in its full capacity range, one can obtain an equation to make predictions applicable to the instrument's output between 1 % and 10 % of its capacity. However, the relative deviation of the calculated and the actual experimental calibration values for a given sub-range should not exceed the maximum relative error values specified by the ISO 376 normative, nor does it have to change the class of the force proving instrument. Since our assessments met these requirements, we are confident with the obtained extrapolation results for the 55 % to 100 % of the devices' maximum capacity.

5. Conclusions

The relative deviation between extrapolated force values was studied via calibration curves from the actual force measurements. This study assessed this behavior within two capacity subranges: one below 10 % and the other between 50 % to 100 % of the force transducer capacity, employing three load cells of 200 N, 500 N, and 100 N capacities. The results show that the relative deviations remain within allowed limits and do not significantly exceed the maximum interpolation error values given in ISO 376 for the three load cells 200 N, 500 N, and 1000 N classes.

The results of this study encourage the use of sub-range calibrations of the force transducer and the extrapolation, based on calibration curves, to predict the whole range of the transducer. It is worth mentioning that the ascertainment of this method requires studies with more force transducer types and a broader set of maximum capacities.

6. Conflict of interests

The authors have no conflict of interests to declare.

References

[1] Ajay Pratap Singh, S. Ghoshal, Development and Metrological Evaluation of a Force Transducer for Industrial Application, *IEEE Access*, 9(3): 33299–33312, 2021.

doi: 10.1109/ACCESS.2021.3060746

[2] Mohamed MI, Hasan EH, Aggag G, Study of Creep Behavior of Load Cells, *Measurement*, 42: 1006–1010, 2009.

doi: 10.1016/j.measurement.2009.03.001

[3] Mohamed MI and Hasan EH, The metrological characteristics of force transducers under loads less than 10 % of its capacity, *IMEKO TC3*, Cairo, 2005.

[4] Hasan EH, et al., Performance evaluation of force transducers, *Indian Journal of Pure and Applied Physics*, 50(2): 86–90, 2012.

[5] Medina N, Validity of extrapolation based on polynomial approximation, *IMEKO 23^rd International Conference*, Helsinki, 2017.

[6] Tegtmeier F, et al. Investigation of transfer standards in the highest range up to 50 MN within EMRP project SIB 63, *XXI IMEKO Congress*, Prague, 2015.

[7] Clegg L, Extending transducer calibration range by extrapolation, *Interface*, 2015.

[8] ISO 376:2011(E), Metallic materials — Calibration of force-proving instruments used for the verification of uniaxial testing machines.

[9] Spiegel MR and Stephens LJ, Theory and problems of statistics, Schaum's outline series, McGRAW-HILL, 2008.

Estudio del error de extrapolación de la ecuación de ajuste de la curva del transductor de fuerza

Resumen: Los laboratorios de calibración a menudo se enfrentan al problema de no poder calibrar el rango total de los transductores de fuerza o a la incapacidad de calibrar rangos menores al 10 % de la capacidad de los transductores de fuerza debido a la falta de disponibilidad de las instituciones para hacer esta calibración. A veces, los laboratorios usan la extrapolación del ajuste de la curva para estimar y predecir el comportamiento de los transductores de fuerza en esos rangos. Esta investigación se ocupa del estudio de los errores de extrapolación para transductores de fuerza con el fin de conocer y estimar lo que se ignora o se pierde en cuanto a exactitud de medida cuando se usa extrapolación en la calibración de transductores de fuerza para cubrir rangos inferiores a 10 % de la capacidad del transductor de fuerza, así como en el rango de 50 % a 100 % de la capacidad. Los resultados de este estudio mostraron que la magnitud del error de extrapolación es muy cercana a la magnitud del error de reproducibilidad de los rangos disponibles para calibrar en el laboratorio.

Palabras Clave: transductor de fuerza; rango de calibración; error de extrapolación; exactitud; incertidumbre.

Estudo do erro de extrapolação da equação de ajuste da curva do transdutor de força

Resumo: Os laboratórios de calibração muitas vezes enfrentam o problema de não conseguir calibrar toda a gama dos transdutores de força, assim como a incapacidade de calibrar faixas inferiores ao 10 % da capacidade dos transdutores de força devido à indisponibilidade das instituições para fazer essa calibração. Às vezes, os laboratórios usam a extrapolação do ajuste da curva para estimar e prever o comportamento dos transdutores de força nessas faixas. Esta pesquisa trata do estudo do erro de extrapolação para transdutores de força, a fim de saber e estimar o que é ignorado ou perdido em termos de precisão da medição quando a extrapolação é usada na calibração de transdutores de força para cobrir faixas inferiores ao 10 % da capacidade do transdutor de força, bem como na faixa de 50 % a 100 % de capacidade. Os resultados deste estudo mostraram que a magnitude do erro de extrapolação está muito próxima da magnitude do erro de reprodutibilidade das faixas disponíveis para calibração em laboratório.

Palavras-chave: transdutor de força; faixa de calibração; erro de extrapolação; exatidão; incerteza

Riham Samir Hegazy Associated Professor at National Institute of Standards (NIS-Egypt). Force and Material Metrology Laboratory. Practical experience in Mechanical testing of materials (Tension, compression, and Hardness), force measurements, calibration of force generating instruments, universal testing machines, hardness testers, hydraulic jacks, calibration of force and torque transducers and material testing metrology. Quality systems, According to ISO 17025, Profiency testing according to 17043 . Practice in technical assessment in the field of force and torque measurements. Practice in evaluating uncertainty budgets in force and torque measurements.

ORCID: 0000-0002-6307-0984

Gouda Mohammad Mahmoud Associated Professor at National Institute of Standards (NIS-Egypt). Force and Material Metrology Laboratory. Practical experience in force measurements, calibration of force generating instruments, universal testing machines, hardness testers, hydraulic jacks, calibration of force and torque transducers and material testing metrology. Quality systems, According to ISO 17025, ISO 17065. Practice in technical assessment in the field of force and torque measurements. Practice in evaluating uncertainty budgets in force and torque measurements.

ORCID: 0000-0001-5986-2455

Magdy Ibrahim Mohamed Professor at National Institute of Standards (NIS-Egypt). Force and Material Metrology Laboratory. Practical experience in force measurements, calibration of force generating instruments, universal testing machines, hardness testers, hydraulic jacks, calibration of force and torque transducers and material testing metrology. Practice in technical assessment in the field of force and torque measurements. Practice in evaluating uncertainty budgets in force and torque measurements.

ORCID: 0000-0001-6402-9302