THE USE OF SIMULINK BLOCK DIAGRAM TO SOLVE MATHEMATICAL MODELS AND CONTROL EQUATIONS

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ABSTRACT

In this paper, the simulink block diagram is used to solve a model consists of a set of ordinary differential and algebraic equations to control the temperature inside a simple stirred tank heater. The flexibility of simulink block diagram gives students a better understanding of the control systems. The simulink also allows solution of mathematical models and easy visualization of the system variables. A polyethylene fluidized bed reactor is considered as an industrial example and the effect of the Proportional, Integral and Derivative control policy is presented for comparison.

Key words: Simulink, block diagram, control systems, mathematical models.

RESUMEN

En este artículo, el diagrama de bloque de simulink es usado para resolver un modelo formado por un conjunto de ecuaciones diferenciales ordinarias y de ecuaciones algebraicas para el control de la temperatura dentro de un tanque calentador con agitador. La flexibilidad del diagrama de bloque de simulink da a los estudiantes una mejor comprensión de los sistemas de control. El simulink también permite solucionar modelos matemáticos y facilita la visualización de sistemas de varias variables. Un reactor de cama de polietileno fluido es considerado como un ejemplo industrial y el efecto en las políticas de control es presentado comparativamente.

Palabras clave: Simulink, diagrama de bloque, sistemas de control, modelos matemáticos.

INTRODUCTION

Chemical plants must operate under known and specified conditions. There are several reasons why this is so, formal safety and environmental constraints must not be violated. Concern for safety is paramount in designing a chemical plant and its control systems. Ideally a process design should be 'intrinsically safe', that is, and plant and equipment should be such so that any deviation, such as an increase in reactor pressure, will itself change operating conditions so that it is rapidly removed. Plants are expensive and intended to make money. Final products must meet customer specifications. Otherwise, they will be unsaleable. Conversely the manufacture of products not meeting the specifications will involve

unnecessary cost. The majority of control loops in a plant control system are associated with operability. Specific flow rates have to be set, levels in vessels maintained and chosen operating temperatures for reactors and other equipment achieved. The top level of process control, what we will refer to as the *strategic* control level, is thus concerned with achieving the appropriate values principally of: production rate, product quality, and energy economy.

A chemical plant might be thought of as a collection of tanks in which materials are heated, cooled and reacted, and of pipes through which they flow. Such a system will not, in general, naturally maintain itself in a state such that precisely the temperature required by a reaction is achieved, a pressure in excess of the safe limits of all vessels be avoided, or a flow rate just sufficient to achieve the economically optimum product composition arise. Notice that this extremely simple idea has a number of very convenient properties. The feedback control system seeks to bring the measured quantity to its required value or setpoint. The control system does not need to know why the measured value is not currently what is required, only that this is so. There are two possible causes of such a disparity:

- The system has been disturbed. This is the common situation for a chemical plant subject to all sorts of external upsets. However, the control system does not need to know what the source of the disturbance was.

- The setpoint has been changed. In the absence of external disturbance, a change in setpoint will introduce an error. The control system will act until the measured quantity reaches its new setpoint. A control system of this sort

should also handle simultaneous changes in setpoint and disturbances.

Modeling and Control Equations

The underlying principle of most process control, however, is already understood by anyone who has grasped the operation of the domestic hot water thermostat:

- The quantity whose value is to be maintained or regulated, e.g. the temperature of the water in a cistern, is measured.

- Comparison of the measured and required values provides an error, e.g. 'too hot' or 'too cold'.

- On the basis of the error, a control algorithm decides what to do.

- Such an algorithm might be:

- If the temperature is too high then turn the heater off. If it is too low then turn the heater on.

- The adjustment chosen by the control algorithm is applied to some adjustable variable, such as the power input to the water heater.

This summarizes the basic operation of a **feedback control system** such as one would expect to find carrying out nearly all control operations on chemical plants and indeed in most other circumstances where control is required.

Simple stirred tank heater

A continuous process system consisting of a well-stirred tank, heater and PID temperature controller is depicted in figure (1).
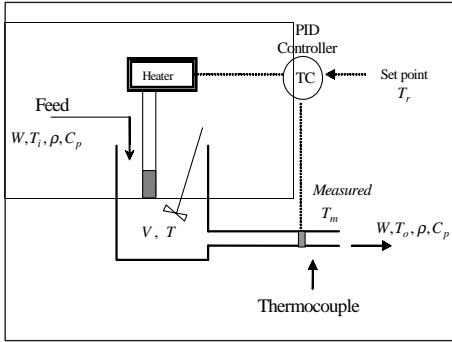
FIGURE 1 Schematic diagram of heating tank

The feed stream of liquid flows into the heated tank at a constant rate of W in kg/min and temperature T_i in °C. The volume of the tank is V in m³. It is desired to heat this stream to a higher set point temperature T_r in°C. The outlet temperature is measured by a thermocouple as T_m in °C, and the required heater input q in kJ/min is adjusted by a PID temperature controller. The control objective is to maintain the temperature input to the thermocouple, $T_o = T_r$ in the presence of a change in inlet temperature T_i which differs from the steady state design temperature of T_{is}.

An energy balance on the stirred tank yields

$$\frac{dT}{dt} = \frac{WC_p(T_i - T) + q}{\rho V C_p} \qquad (1)$$

With initial condition $T = T_r$ at $t = 0$ which corresponds to steady state operation at the set point temperature T_r. The thermocouple for temperature sensing in the outlet stream is described by a first order system plus the dead time τ_d which is the time for the output flow to reach the measurement point. The dead time expression is given by

$$T_o(t) = T(t - \tau_d) \qquad (2)$$

The effect of dead time (τ_d) may be calculated for this situation by the Padé approximation which is a first order differential equation for the measured temperature.

$$\frac{dT_o}{dt} = \left[T - T_o - \left(\frac{\tau_d}{2} \right)\left(\frac{dT}{dt} \right) \right] \frac{2}{\tau_d}$$

I. C. $T_o = T_r$ at $t = 0$ (steady state) $\qquad (3)$

The above equation is used to generate the temperature input to the thermocouple, T_o.

The thermocouple shielding and electronics are modeled by a first order system for the input temperature T_o given by

$$\frac{dT_m}{dt} = \frac{T_o - T_m}{\tau_m}$$

I. C. $T_m = T_r$ at $t = 0$ (steady state) $\qquad (4)$

where the thermocouple time constant τ_m is known. The energy input to the tank, q, as manipulated by the proportional/integral (PID) controller can be described by

$$q = q_s + K_c(T_r - T_m) + \frac{K_c}{t_I}\int_0^t (T_r - T_m)dt + K_c t_d \frac{d}{dt}(T_r - T_m) \quad (5)$$

where K_c is the proportional gain of the controller, t_I is the integral time constant or reset time and t_d is the derivative time constant. The q_s in the above equation is the energy input required at steady state for the design conditions as calculated by

$$q_s = WC_p(T_r - T_{is}) \qquad (6)$$

The integral in Equation (5) can be conveniently be calculated by defining a new variable as

$$\frac{d}{dt}(err) = T_r - T_m$$

I. C. $err = 0$ at $t = 0$ (steady state)   (7)

Thus Equation (7) becomes

$$q = q_s + K_c(T_r - T_m) + \frac{K_c}{t_I}(err) + K_c t_d \frac{d}{dt}(T_r - T_m) \quad (8)$$

RESULTS AND DISCUSSION

Detailed of model equations are shown in the simulink block diagram of figure 2 where detailed model equation is being solved. The PID controller equations of figure 2 are replaced by a PID simulink block diagram and are shown in figure 3. Operating parameters are listed in table 1.
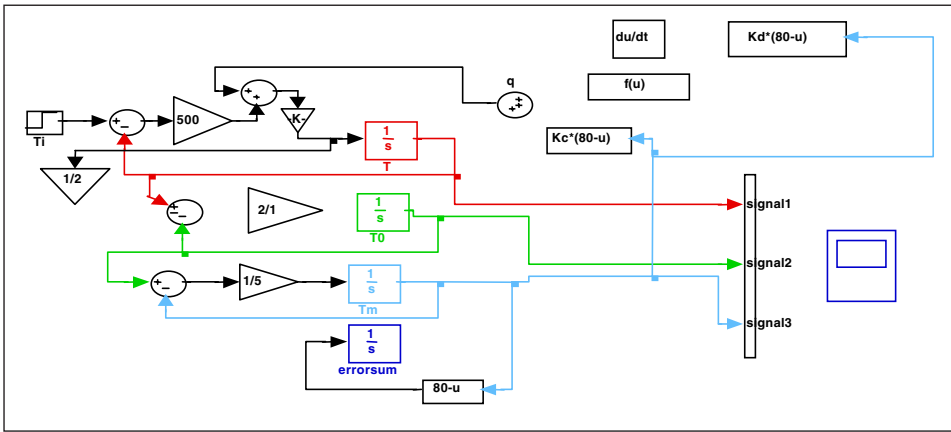

FIGURE 2. Schematic diagram of simulink flowsheet of the model equations.
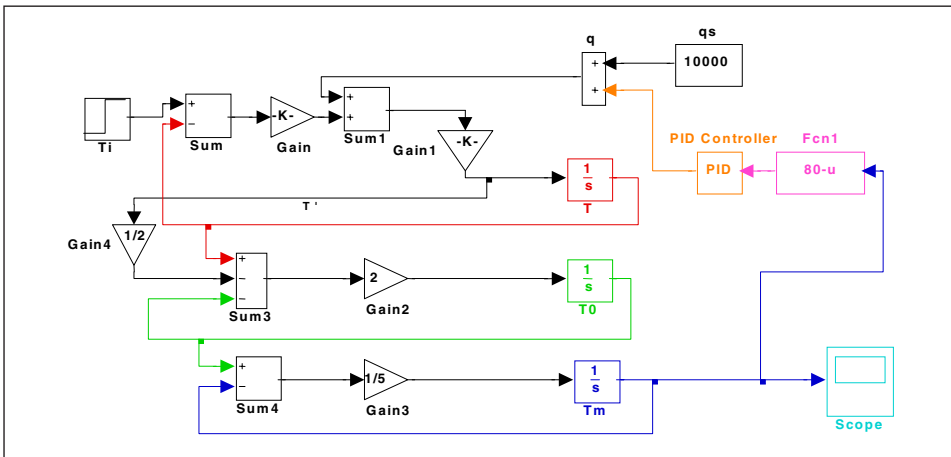

FIGURE 3. Schematic diagram of SIMULINK flowsheet of the PID control tool box.

$\rho V C_p = 4000 \, kJ / ^o C$	$WCp = 500 \, kJ / \min .^o C$
$T_{is} = 60 \,^o C$	$T_r = 80 \,^o C$
$\tau_d = 1 \min$	$\tau_m = 5 \min$
$K_c = 50 \, kJ / \min \,^o C$	$t_I = 2 \min, \; t_d = 1 \min$

The controlled variable is the heat measured temperature T_m, while the manipulated variable is the power input q to the heater. Values of K_c, t_I , t_d should be given in the MATLAB workspace before running the simulink block diagram. Figure 4a shows an open loop system; under a certain disturbance of inlet feed temperature, the heater temperature drops from 80 to 60ºC. Implementing a Proportional controller where K_c =500 still could not bring the well stirred tank heater to its set point temperature (fig. 4b). Addition of an integral action where the integral time constant, t_I =2 brings the heater back to its set point (fig. 4c). Increasing t_I to 20 again took the tank temperature away from the set point temperature meaning that the increase in the integral time constant is not desired (fig. 4d). An addition of derivative time constant t_d did not make much change (fig. 4e, f).
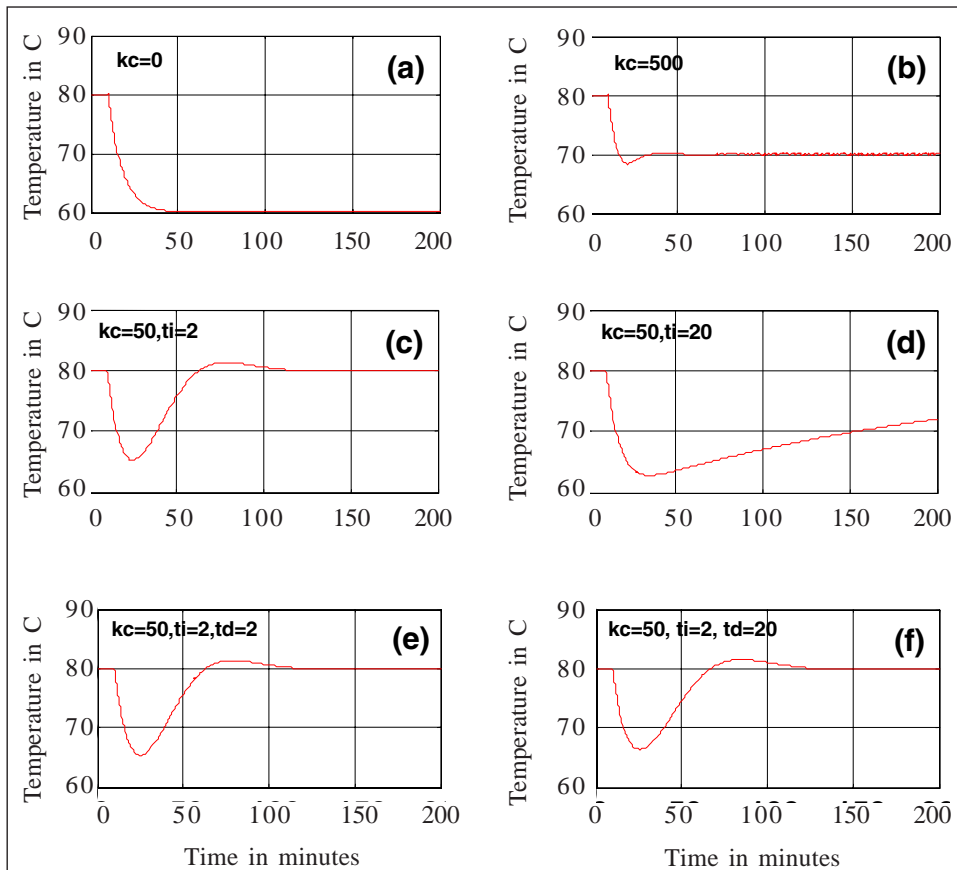

FIGURE 4. Effect of controller gain on tank temperature.

The Industrial UNIPOL process for polyethylene production

Polyethylene is made in polymerization reactions in the presence of a catalyst. Both the reactor technology and the catalyst technology are patented, and both Dow and Carbide are leading developers of reactor technology. Carbide's reactor technology, called Unipol (fig. 5), is the world's most widely licensed polyethylene process technology. The other significant licensed LLDPE (Linear Low Density Polyethylene) process technology is Innovene, owned by BP. Both UNIPOL and Innovene make polyethylene in a process in which ethylene is in a gaseous form during polymerization (gas phase). The large majority of LLDPE reactor operates in gas phase rather than in liquid phase. Polyethylene achieves its preeminent position in thermoplastic industries due to a remarkable catalyst and gas-solid fluidized bed technology (Choi and Ray 1985, Debling *et al*., 1994). In addition, by employing the Ziegler-Natta catalyst, the process can operates at relatively low pressure and temperature (20 atm and 100°C) compared to the conventional process (2000 atm, 200°C). However, improper control of the process parameters especially the catalyst injection rate, raw material feed temperature and superficial gas velocity may lead to temperature runaway and clusters formation in the reactor. Subsequently plant has to shut down for cleaning purpose. In addition, the situation becomes worse when the reactor bed temperature exceeds the polyethylene softening point (~400 K), where the solid particles tends to agglomerate and may form huge chunk in the reactor.
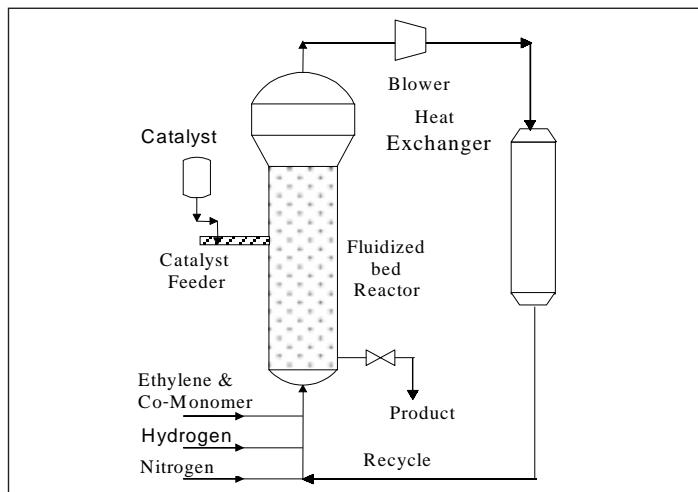

FIGURE 5. Schematic diagram of UNIPOL process

Modeling of polyethylene gas phase reactor (UNIPOL process)

Detail of the modeling equation can be found elsewhere (Debling *et al*., 1994, McAuley *et al.,* 1994). Solution of model equation using SIMULINK block diagram is shown in figure 6.
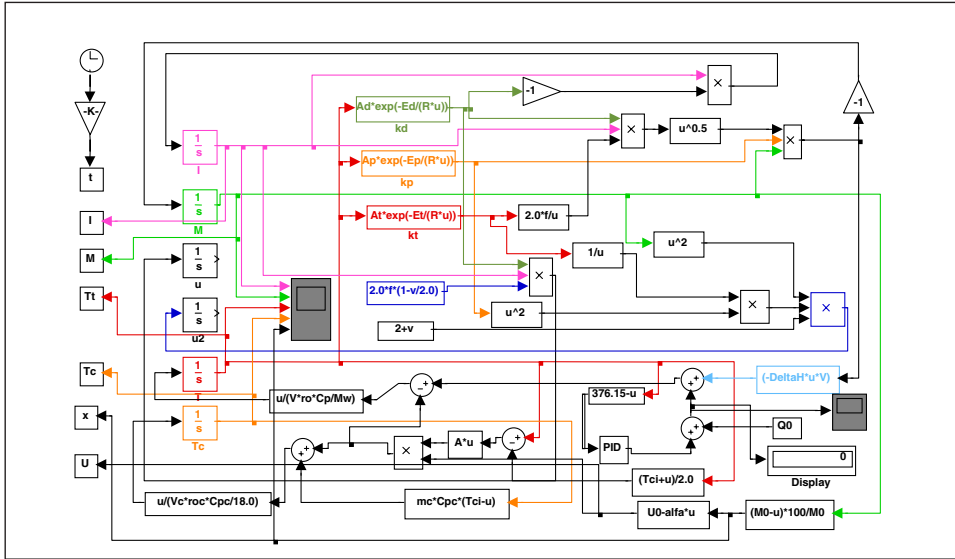
FIGURE 6. SIMULINK flowsheet to the well mixed model for polyethylene fluidized bed reactor

RESULTS AND DISCUSSION

The effect of a step change in reactor feed temperature on the dimensionless reactor temperature. It is clear that the system was stable before a step change in the feed temperature takes place. Once the step change in feed temperature occurs the temperature loses its stability and an oscillatory behavior above polymer melting point occurs.

Implementing a proportional controller where the control variable is the reactor bed temperature and the manipulated variable is the heat exchange cooling water temperature, the system regain its stability but still with some offset (fig. 7b). With an introduction of an integral controller along with proportional gain the reactor returns to its set point (fig. 7c).
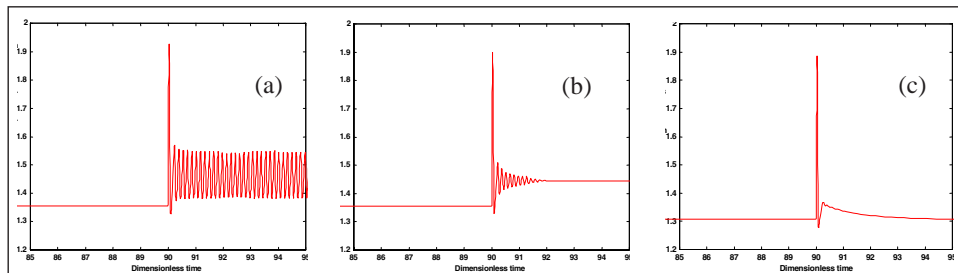
(a) (K_c=0.0, K_I=0.0), (b) (K_c=0.50, K_i=0.0), (c) (K_c=0.50, K_I=0.10)

FIGURE 7. Effect of controller gain fluidized bed reactor temperature

CONCLUSION

- At low values of integral action (reset) time and low proportional bands, the system is prone to significant oscillation. This large amount of oscillation means that the control tends towards on/off control.

- As the proportional band is increased the oscillation in the system becomes less and the process is successfully returned to the setpoint (recall with proportional only control, there was an "offset"). However, the process is away from the setpoint for a relatively long period of time.

- As the integral action (reset) time is increased, so the system is returned to the setpoint more quickly. However, at high proportional bands and high integral action (reset) times the system again takes a long time to return (and in fact it may not) to the setpoint as the system is "over damped".

- The main advantage of PI control is that offset is eliminated. The combination of correct proportional bands and a correct integral action (reset) time produces a quick response to a disturbance in the process that returns the process to the setpoint without offset.

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