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A NEW DIMENSION OF INTERDISCIPLINARITY: ECONOPHYSICS

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ABSTRACT

The paper aims to offer an overview on some recent application of statistical physics methods to economic and financial problems, field known today as "econophysics".

The second part introduces an example of microscopic modeling, namely the financial crashes seen as second order phase transitions. Although the model proposed is minimal —the Ising model in Bragg-Williams approximation— it leads to a discontinuity in the shape of the specific heat similar to the discontinuities of stock market indices during a financial crash.

RESUMEN

El trabajo propone una corta mirada general sobre un dominio nuevo de las ciencias interdisciplinarias basado en la aplicación de los métodos de la física teórica en el estudio de los fenómenos y de los procesos económicos y sociales.

La segunda parte del trabajo presenta una aplicación concreta de este género de modelación, es decir, las caídas financieras modeladas por intermedio de los métodos de la termodinámica estadística. Los datos están tomados de la caída financieras de New York Stock Exchange de 19.10.1987.

1. INTRODUCTION. ECONOPHYSICS – A NEW AREA FOR STATISTICAL PHYSICS?

The word "econophysics" was introduced by H.E. Stanley to describe the large number of papers written by physicists in the last ten years on problem of (stock) market, the growth of companies and related economic questions (J.P. Bouchaud and M. Potters, 2000; R.N. Mantegna and H.E. Stanley, 2000; H. Levy *et al.*, 2000). The first econophysics model published by physicists in physics journals were those published by Mantegna (R.N. Mantegna, 1991), but clearly physicists did not bring the physical methods to the economic science. For example, a Monte Carlo simulation of a market was already published in 1964 by Stigler (G.J. Stigler, 1964) from the

Chicago economics school and economy Nobel laureate Markowitz (G.W. Kim and H.M. Markowitz, 1989) published with Kim a model for the 1987 crash on Wall Street with two types of investors similar to many later models of physicists; economist Lux (T. Lux, 1996) has cited the work of physicists like Haken; and some other articles were published in between. Thus, the question in the title of the section can be answered with a clear "no". The field is not new. What physicists did was to enlarge the number of people using these methods, to get better data, or to use very specific physics results less known in economics.

The econophysicist D. Stauffer compares in one of his papers "econophysics" with the "discovery" of America by Columbus half a

millennium ago: Other people came thousands of years earlier from Asia, and the Normans settled for some time in Vinland, now in Northern Newfoundland. But none of them informed about these findings a more widespread medium, while the voyage of Columbus really changed life in both America and Europe. In this sense the econophysicists are like Columbus, not really knowing they are doing but nevertheless doing something important.

G. Soros pointed out in 1994 in his "Alchemy of Finance" the inadequacy and the inefficiency of the existing theories about the behavior of stock prices. Until the last decade the theoretical economics was dominated by pure mathematics characterized by ridiculous lemma/theorem style, little effort to compare theoretical predictions to "experiment" (say, prices from real stock markets) and the fact that bulk of papers are inaccessible and of no interest to "experimentalists" - practitioners of the field. However the pure mathematics has contributed to economics through the Game Theory approach (the concept of Nash equilibrium where no player can improve on his/her strategy and the supposition of the perfect rationality of all players) and the phenomenology of stock price fluctuations that are postulated to be Gaussian and subject to the Efficient Market Hypothesis: all correlation's are arbitrated away. There are many observations in disagreement with these suppositions: First, short term fluctuations are non-Gaussian; The second, price increments are correlated (the magnitude of price fluctuations has long temporal correlation's); The third, strategies used by trades are correlated as manifested by herd effect.

From this point physics is called to bring its contributions. The physicists can work with empirical data and construct phenomenological theories. Also, statistical physics field has useful approaches to deal with collective dynamics composed of many interacting parts.

As the traditional physics, econophysics can be divided into experimental and theoretical, the first trying to analyze real data from real markets and to make sense of them, the second trying to find microscopic models which give for some quantities good agreement with the experimental facts. In the last years econophysics has matured enough to allow some applications, their field being called econo-engineering: the financial applications want to advise banks and brokers how to estimate risks and demand proper fees to balance these risks.

From the large field of econophysics, we have selected for present paper only the problem of financial crashes modelling. In the next section we describe briefly the analogies between these crashes and the thermodynamic phase transitions.

Starting from here, in Section 3 we propose a simple and suggestive model to explain the critical points arising in the stock market behavior. The last section draws some conclusions.

2. THE STOCK MARKET CRASHES SEEN AS PHASE TRANSITIONS

The similarities between an economic crash and a phase transition have been underlined first in independent works by Sornette (D. Sornette *et al.*, 1996) and Feigenbaum (J.A. Feigenbaum and P.G.O. Freud, 1996). Since then, many other articles were published by physicists on this problem (D. Sornette and A. Johansen, 1997, 1998; N. Vandewalle *et al.*, 1998a; L. Laloux *et al.*, 1999; A. Johansen *et al.*, 2000; T. Kaizoji, 2000). Recently, the problem of the noise induced transitions was approached by Gligor (M. Gligor, 2001).

In the stock market the price changes are subject to the law of demand and supply, that the price rises when there is excess demand, and the price falls when there is excess supply. It seems natural to assume that the price raises if the number of the buyers exceeds the number

of sellers because there may be excess demand, and the price falls if the number of sellers exceeds the number of the buyers because there may be excess supply. A crash happens when a large group of agents place sell orders simultaneously. This group of agents must create enough of an imbalance in the order book for market makers to be unable to absorb the other side without lowering prices substantially.

From the opening on Wednesday, 23 October 1929, to the closing of Tuesday, 29 October 1929, the New York Stock Exchange lost almost 30% of its value. In similar fashion, major index of market valuation in the United States declined by 30% or more from the opening on 14 October 1987 to the market close on October 19 and in addition all major world markets declined substantially in the following month, in contrast with the usual modest correlation's of returns across countries. More recently on 23 October 1997 and the following week the Hong Kong Stock Exchange exhibited a crash of similar proportion (24%).

Even though the market crashes are considered as highly rare and unpredictable events, it should be noted that they take place systematically during periods of generalized economic euphoria.

These periods are characterized by the formation of "speculative bubbles" in the space of the stock market prices (Gligor and Ignat, 2001) (the brokers overvalue some assets; these stocks are bought in order to resell them later on, not for their realistic value).

Now, let us recall some features of the phase transitions. The processes of boiling-condensation, melting-freezing, and congelation-sublimation involve changes in entropy (S) and volume (V). According to Ehrenfest, such transitions are classed as changes of phase of the first kind (or the first order): whereas the Gibbs function remains constant during a

change of phase, its first derivatives S and V change abruptly. A distinction of a phase change of the first kind is that the new phase is formed gradually: The fraction of the new phase gradually increases as more of the latent heat of the respective phase change is put into or withdrawn out of the system.

There exist phase changes of higher orders in which S and V retain their values constant during a phase change. In the process T (temperature), p (pressure) and U (internal energy) likewise remain unchanged. If, during a phase change, c (the specific heat) and the others calorific coefficients are incremented or decremented by a finite amount, such phase transitions are called those of the second kind (or of the second order) because each of these quantities can be defined as a second derivative of the Gibbs function. An example is the transition from the state of superconductivity to the normal state in the absence of a magnetic field.

An example of a higher order phase change is the lambda phase change, so called because the $c = f(T)$ curve looks like the Greek letter "lambda" (Figure 1- continuous line). Examples of the lambda phase change are the transition from the ferromagnetic to the paramagnetic state or from He I (the ordinary, viscous state of helium) to He II (a peculiar state called superfluid). The lambda transition proceeds without an abrupt change in density (the density curve has a quiet peak around the λ -point), without evolution or absorption of latent heat, and is accompanied by a sudden change in the specific heat (A. Arkharov *et al.*, 1981).

Fig. 1 and Fig. 2 show several similarities between the physical complex systems and stock markets. We can ask now what is the key of these similarities. Our Ariadne's thread is that complex systems often reveal more of their structure and organization in highly stressed situations than in equilibrium. This point of view is influenced by the concept of

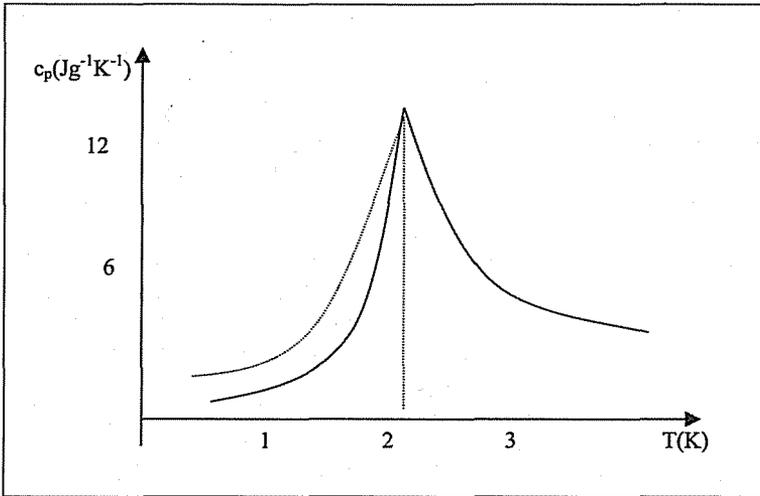


FIGURE 1. The specific heat of Helium near the critical point [18] (continuous line); The specific heat variation according the Ising model in Bragg- Williams approximation [19] (dashed line).

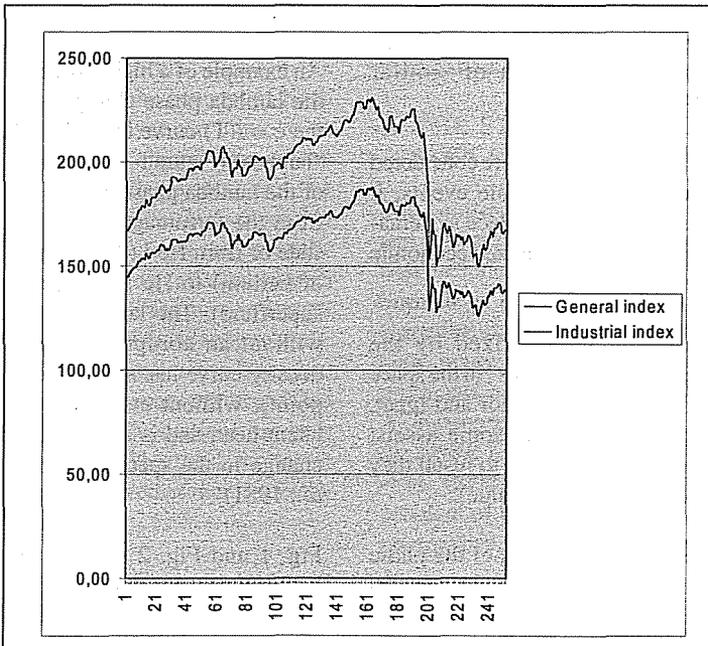


FIGURE 2. The Dow Jones Average (General or Composite and Industrial) Indices between 4 Jan and 28 Dec 1987.

criticality developed in statistical physics in the last 30 years in order to describe a class of cooperative phenomena, such as magnetism and melting, and our hypothesis is that the stock market behaves as a driven out-of-equilibrium many-body system. In the next section we would like to defend the thesis that the crashes have fundamentally similar origins, which must be found in the collective organization of the market traders leading to a regime known as a “critical” point.

3. MICROSCOPIC MODELLING

As we have claimed in the previous section, a stock market crash happens when a large group of agents place sell order simultaneously. One curious fact is that the agents in this group typically do not know each other. They did not convene a meeting and decide to provoke a crash. Nor do they take orders from a leader. In fact, most of the time, these agents disagree with one another, and submit roughly as many buy orders as sell orders (when a crash does not happen). The key question is: by what mechanism did they suddenly manage to organize a coordinated sell-off?

Accordingly Sornette and Johansen (D. Sornette and A. Johansen, 1998), we propose the following answer: all the traders in the world are organized into a network (of family, friends, colleagues, etc.) and they influence each other *locally* through this network. Each of them is directly connected with k nearest neighbors, and there are only two forces that influence his opinion: (a) the opinions of these k people; (b) an idiosyncratic signal that he alone receives. Our working assumption is that agents tend to *imitate* the opinions of their nearest neighbors, not contradict them. Clearly, the force (a) will tend to create order, while force (b) will tend to create disorder. The main story that we are telling on is the fight between order and disorder. As far as asset prices are concerned, a crash happens when order wins (every agent has the same opinion: selling), and normal times are when

disorder wins (buyers and selling disagree with each other and roughly balance each other out). This is exactly the opposite of the popular characterization of crashes as time of chaos.

Now, let us consider the traders network as an arrangement of N fixed points forming a n -dimensional periodic lattice ($n = 1, 2, 3$). Each site of this lattice has attached a number S_i ($i = 1, \dots, N$) taking only two values: $S_i = +1$ (the spin “up”; selling option) and $S_i = -1$ (the spin “down”; buying option). A set of numbers $\{S_i\}$ specifies a configuration of the whole system. The energy corresponding to this configuration in the absence of external fields is:

$$E \{S_i\} = - \sum_{\langle ij \rangle} \epsilon_{ij} S_i S_j \quad (1)$$

where $\langle ij \rangle$ appoints a pair of nearest spins and ϵ_{ij} is their interaction energy. Because $\langle ij \rangle$ and $\langle ji \rangle$ are not distinct, the sum will have $? N/2$ terms, with $? =$ the number of nearest neighbors of a given spin. In an established configuration, we call:

N_+ = the total number of spins “up”;

N_- = the total number of spins “down”.

Each pair of spins from the sum belong to one of the kinds: $(++)$, $(--)$, $(+-)$, the last being no distinct from $(-+)$. The corresponding number of pair will be N_{++} , N_{--} , N_{+-} . These numbers are not independent. The relations between they can be established as follows:

a) we link by lines a spin “up” with its nearest neighbors.

Repeating for all the spin “up” we obtain $\tilde{a} N_+$ lines.

b) the number of double lines will be N_{++} and the number of simple lines, N_{+-} .

Therefore $\tilde{a} N_+ = 2N_{++} + N_{+-}$

interchanging the indices “+” and “-” we have:

$$\tilde{a} N_- = 2N_{--} + N_{+-}$$

The equations:
$$\begin{cases} \tilde{a} N_+ = 2N_{++} + N_{+-} \\ \tilde{a} N_- = 2N_{--} + N_{-+} \\ N_+ + N_- = N \end{cases} \quad (2)$$

have the solutions:
$$\begin{cases} N_+ = N - N_- \\ N_{+-} = \tilde{a} N_+ - 2N_{++} \\ N_{-+} = \tilde{a} N_- - 2N_{--} \\ -\gamma N_+ \end{cases} \quad (3)$$

so we can write:

$$\sum_{(ij)} S_i S_j = N_{++} + N_{--} - N_{+-} - N_{-+} = 4N_{++} - 2\gamma N_+ + \frac{\gamma}{2} N \quad (4)$$

Note that although the system configuration depends on N numbers, the energy of a given state depends only on two numbers: N_+ and N_{++} (We can consider $\tilde{a}_{ij} = \tilde{a} = \text{constant}$). The number N_+/N measures the “long range” order, while $N_{++}/(N/2)$ measures the “short range” order. The reason of this terminology is the following: Having given a random distribution of spins and knowing that a certain spin is “up”, the number $N_{++}/(N/2)$ is the fraction of nearest neighbors having the spin “up”, thus involving a local correlation between spins: The other number, N_+/N , does not imply correlation’s between the nearest neighbors, but represents the fraction of spins from all the lattice having the orientation “up”. We define the long-range order parameter L and the short-range order parameter σ through the relations:

$$\frac{N_+}{N} = \frac{1}{2}(L + 1) \quad ; \quad (-1 \leq L \leq 1) \quad (5)$$

$$\frac{N_{++}}{(1/2)\gamma N} = \frac{1}{2}(\sigma + 1) \quad (-1 \leq \sigma \leq 1)$$

Replacing into Eq.(4) we get:

$$\sum_{(ij)} S_i S_j = \frac{1}{2}\gamma N (2\sigma - 2L + 1) \quad (6)$$

and the energy per spin, from Eq.(1), becomes:

$$\frac{1}{N} E(L, \sigma) = -\frac{1}{2} \epsilon \gamma (2\sigma - 2L + 1) \quad (7)$$

The Bragg – Williams approximation (BWA) can be summarized into the assertion that there is no short range order besides that involved by the long range order. Concretely it assumes that:

$$\frac{N_{++}}{\frac{1}{2}\gamma N} = \left(\frac{N_+}{N}\right)^2 \quad , \text{ or: } \sigma = \frac{1}{2}(L + 1)^2 - 1 \quad (8)$$

Thus, the BWA leads to the energy per spin:

$$\frac{1}{N} E(L) = -\frac{1}{2} \epsilon \gamma L^2 \quad (9)$$

In literature, there are several tools for evaluating the parameter L . The simplest way is based on the partition function (see K. Huang, 1963 for more details), leading to the transcendental equation:

$$L = \tanh\left(\frac{\gamma \epsilon L}{kT}\right) \quad (10)$$

where k is Boltzmann’s constant and T is temperature (K). Eq.(10) has non-trivial solutions only for:

$$\frac{\gamma \epsilon}{kT} > 1 \quad , \text{ i.e. } T < T_c = \frac{\gamma \epsilon}{k}$$

In this case there is a spontaneous orientation of spins (the system is “ferromagnetic”). The solutions¹ $\pm L_0$ can be approximated by:

$$L_0 \approx 1 - 2 \exp\left(\frac{-2T_c}{T}\right)$$

and, finally, the specific heat will be given by:

$$\frac{1}{Nk} c(T) = -\frac{\epsilon \gamma}{2} \frac{d}{dT} L_0^2 \quad \text{for } T < T_c \quad (11)$$

having the shape dashed in Fig. 1.

For: $\frac{\gamma \epsilon}{kT} < 1 \quad , \quad \text{ i.e. } T > T_c = \frac{\gamma \epsilon}{k}$

¹ The degeneracy $L = \pm L_0$ is due to non-existing an intrinsic difference between the orientations “up” and “down” of spins in absence of external fields.

the system has not spontaneous magnetization. The specific heat vanishes indicating the absence both short-range and long-range order.

The experimental curve is also plotted in Fig. 1 (the continuous line).

In spite of its gross simplifications, the model lead to the theoretical shape of the specific heat exhibiting a jump at T_c . A similar jump can be noted for the Dow Jones Average indices behaviors on 19 October 1987 ("The Black Monday") – Fig. 2.

4. CONCLUSIONS

Stock markets are fascinating structures with analogies with arguably the most complex dynamical system found in the Natural Sciences, i.e., the human mind. The simplified model suggested in the previous section can at most point out the similarities between stock market and physical systems extreme behaviors. A lot of models improve this point of view carrying out moreover (a first improvement is suggested by Gligor and Ignat (M. Gligor and M. Ignat, 2001) by using a more complex version of the Ising model, namely the Bethe – Peierls approximation), but their mathematical approach exceeds the aims of present paper. We will nevertheless focus on some natural questions.

The first: The stock market prices depend on t (time), while the specific heat depends on T (temperature). The two parameters have completely different physical meanings. Their association could appear forced, but certainly there is a critical time t_c in the evolution of financial systems such as there is a critical temperature T_c for phase transitions. In both cases, the essential fact is to find a relevant parameter (no matter its nature) that measures the deviation of the system from its equilibrium state. We have considered to be T this parameter.

The second question: Are the financial crashes predictable? The answer seems to be affirma-

tive, taking into consideration the behavior of the stock market prices before the crash (see Fig. 2). These prices follow log-periodic oscillations (proportional to $\sin((const) \log t)$) that are known from some exactly solved models and simulations of similar models. These log-periodic oscillations have been seen in many stock markets before the crashes and the large number of fits makes them more trustworthy. A clear example: The Liège group predicted a crash to happen before the end of November 1997 and this was reported in a magazine dated mid-September 1997 (H. Dupuis, 1997; N. Vandewalle *et al*, 1998b). A crash happened at the end of October of that year; this is not bad for the beginning.

Now we could ask: Are the financial crashes avoidable? What would happen if in the years to come many more such successful predictions would be published: Finally, the market practitioners would believe them. If a prediction is made and believed that the market will remain calm for one month and then crash, this prediction would destroy itself since investors would then try to sell before the crash, thus making the prices fall earlier. If not all investors believe and act simultaneously, then the crash would be smoothened out which would make life better for most.

It would be nice if hundred years from now, textbooks of finance would state that around the year 2000 some physicists developed methods to predict and prevent major crashes on stock markets.

And if this is too much to ask for, at least we may have contributed to better estimates of the probability of larger price changes, thus reducing the number of bankruptcies.

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